Below, the same questions are given in English:

NOTE: -On the fourth page of the exam paper, there are some useful equations presented.

1. Robot off-line programming method: Working principle? Benefits compared to traditional robot on-line programming? Calibration tasks/problems related to the method? (4 points)

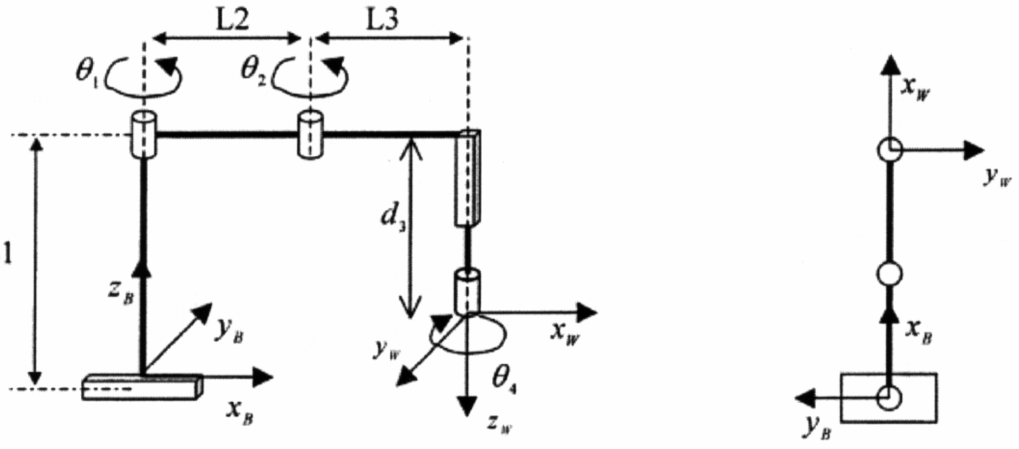
2. In figure 1 the kinematic structure of a four degree-of-freedom SCARA robot is shown. The first two joints are rotational (shoulder and elbow joints move the arm on a plane), then a prismatic joint follows, which moves the tool up and down. And finally, in the kinematic chain, a rotational joint adjusts the orientation of the tool, for example, to grasp objects on a pallet, oriented parallel to the xy-plane of the B-frame.
   a) Number and mark in the figure the link-frames required for constructing the direct kinematic transformation of the manipulator for describing the wrist frame (W) with respect to the base frame (B). Also draw into the figure and give in a table the link parameters and variables (i.e. Denavit-Hartenberg parameters). Define also the corresponding homogenous link transformation matrices. (4p)
   b) Define the transformation equations for describing the orientation of the wrist frame (W) by means of the X-Y-Z fixed angles (i.e. Roll, Pitch Yaw angles) as a function of robot joint angles. (4p)

3. In figure 2 a two decree-of-freedom manipulator is shown in its home/zero position, the first dof is a rotational joint (controlling the orientation of the upper link on the horizontal plane), θ, and the second dof is a translational joint (controlling the length of the upper link), d. (The upper link is above the negative y-axes of the 0-frame when the control angle θ has a zero value). Find the inverse kinematic transform for the manipulator. Describe also, for which of the x,y,z-positions of the origin of the (W) frame a reachable inverse kinematic solution exists (answer , for example, in the form of equations or inequalities)? (4p)

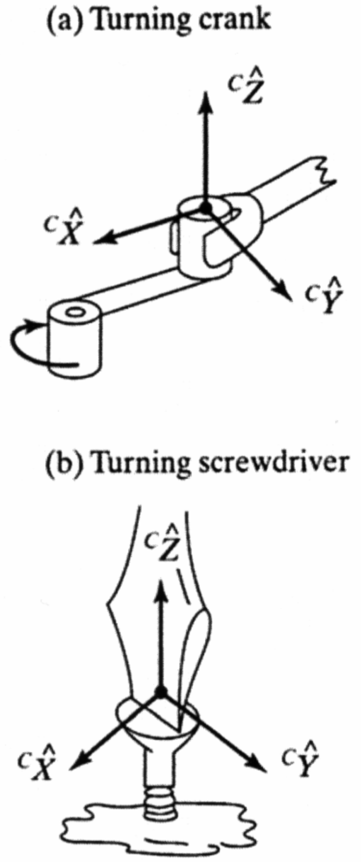
4. Natural constraints and corresponding controllable parameters (artificial constraints) in the example tasks of figures 3a) and 3b). In other words, explain natural constraints and corresponding control values (linear/angular velocity or force/moment) with respect to each of the six degrees of freedom of the task frame? (4p)

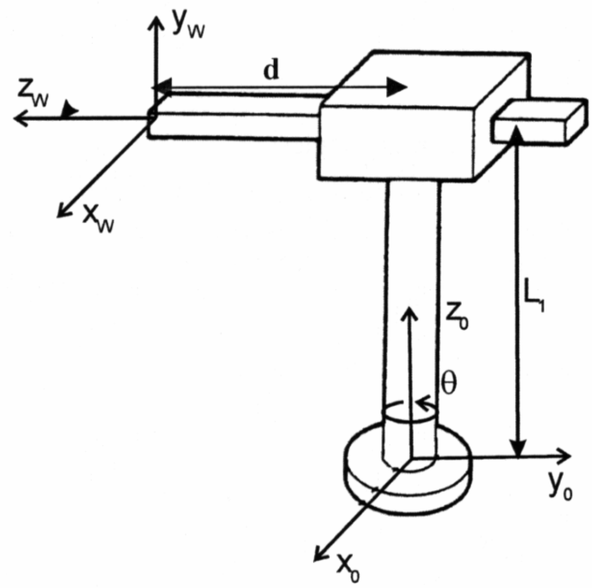
5. In figure 4. the possible routes from start point (Start) to goal point (Goal) are shown. The circles represent intermediate goals through which the robot must move. The distances between two reachable intermediate goals are represented with numbers attached to the connecting lines. Describe in detail how to find the shortest path from 'S' to 'G' with the 'A*' search method ? (4p)

6. A single-link manipulator with a rotary joint is motionless in position $\theta = -5$ degrees at time $t = 0$. It is desired to move the joint in a smooth manner to $\theta = 80$ in 4 seconds i.e. $\theta_0 = -5$, $\theta_f = 80$, $t_f = 4$. Give the equations for **location**, **speed** and **acceleration** of the manipulator joint when the equation for the location is given in form of a cubic polynomial:
   $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$, where
   $$a_0 = \theta_0,\ a_1 = 0$$
   $$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$
   $$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$
   In addition, plot/sketch the position, velocity and acceleration of the joint as a function of time $t \in [0,4]$.



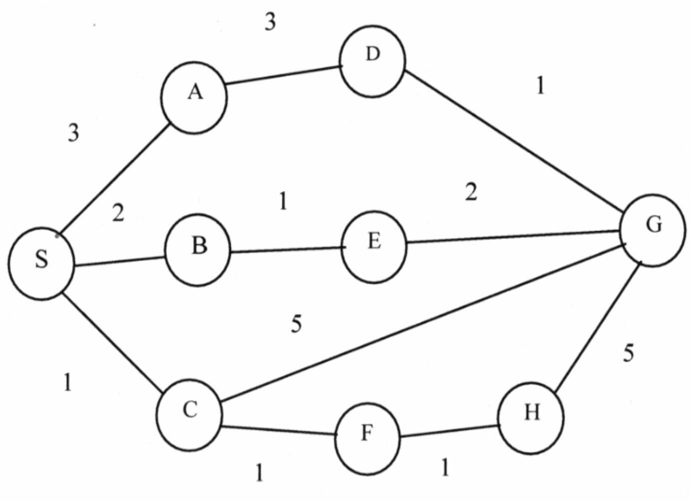
Kuva/Figure 1





Kuva/Figure 2

Kuva/Figure 3



Kuva/Figure 4

Rotation about the principal axes: $$R_X(\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad \text{(A.1)};$$

$$R_Y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \text{(A.2)} \quad R_Z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \text{(A.3)}$$

Homogenous transform:

$${}^A_BT=\left[\begin{array}{c|c} {}^A_BR & {}^AP_{BORG} \\ \hline 0\ \ 0\ \ 0 & 1\end{array}\right]=\begin{bmatrix}r_{11} & r_{12} & r_{13} & p_x\\ r_{21} & r_{22} & r_{23} & p_y\\ r_{31} & r_{32} & r_{33} & p_z\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \text{(2.1)+(2.2)+(2.19)}$$

X-Y-Z fixed angles:

$${}^A_BR_{XYZ}(\gamma,\beta,\alpha)=R_Z(\alpha)R_Y(\beta)R_X(\gamma) \qquad (2.63)$$

$$=\begin{bmatrix}c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma & 0\\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma & 0\\ -s\beta & c\beta s\gamma & c\beta c\gamma & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (2.64)$$

$${}^A_BR_{XYZ}(\gamma,\beta,\alpha)=\begin{bmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix} \quad (2.65)$$

$$\begin{aligned}\beta &= a\tan 2\left(-r_{31},\sqrt{r_{11}^2+r_{21}^2}\right)\\ \alpha &= a\tan 2\left(\frac{r_{21}}{\cos\beta},\frac{r_{11}}{\cos\beta}\right)\\ \gamma &= a\tan 2\left(\frac{r_{32}}{\cos\beta},\frac{r_{33}}{\cos\beta}\right)\end{aligned} \qquad (2.66)$$

Link transformation:

$${}^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i)$$

$$=\begin{bmatrix}c\theta_i & -s\theta_i & 0 & a_{i-1}\\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i\\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \text{(3.4)+(3.6)}$$

Inverse of a homogenous transform:

$${}^A_BT^{-1}={}^B_AT=\left[\begin{array}{cc} {}^A_BR^T & -{}^A_BR^T\,{}^AP_{BORG} \\ \hline 0\ \ 0\ \ 0 & 1\end{array}\right] \qquad (2.45)$$