AS-74. 3125 Optimal, Adaptive and Robust Control
Exam 29. 8. 2012

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problems have been evaluated.

5 problems.

1. Consider the first order process

$$\frac{dy}{dt} = -ay + bu \quad \text{(parameters } a \text{ and } b \text{ are unknown constants)}$$

and the reference model

$$\frac{dy_m}{dt} = -a_m y_m + b_m r \quad (a_m \text{ and } b_m \text{ are constants set by the designer})$$

Design an adaptive feedback control law to the system, when the parameter adjustment is designed by the MIT rule. Present a "Simulink-like" figure presenting the closed-loop system.

2. Consider the process

$$\dot{x}(t) = u(t)$$

and the criterion to be minimized

$$J = \int_0^1 \left(x^2(t) + u^2(t) \right) dt$$

Find the optimal control law by using

a. Calculus of variations, with the initial state $x(0) = x_0$ and final state $x(1) = x_f$

b. Calculus of variations, with the initial state $x(0) = x_0$ and final state $x(1) =$ free

Hint for part b: $\frac{\partial g}{\partial \dot{x}}(x^*(t_f), \dot{x}^*(t_f), t_f) = 0$

3. Write the nonlinear autonomous system

$$\ddot{y}(t) + \dot{y}(t) + y^3(t) = 0$$

in state-space form. Consider

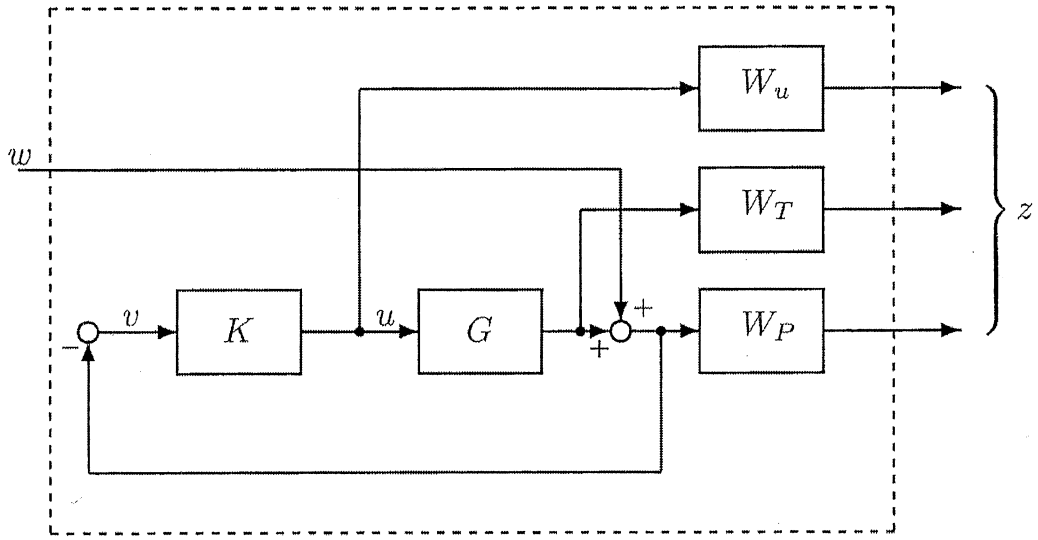
$$V(t) = ax_1^4(t) + bx_1^2(t) + cx_1(t)x_2(t) + dx_2^2(t)$$

and choose the parameters a, b, c, d such that

$$\dot{V}(t) = -x_1^4(t) - x_2^2(t)$$

Investigate the stability nature of the equilibrium point at the origin.

4. Consider the following control configuration



where G is the process, K the controller, Wu, W_T and W_P are weights.

a. Present the system in the general control configuration (calculate P).
b. Close the loop, i.e. calculate N from $z = Nw$.

5. Explain shortly the following concepts (and their meaning in control)

a. Gain scheduling
b. Direct and indirect adaptive algorithms
c. Pontryagin's principle.
d. Positive and strictly positive transfer functions
e. MΔ structure in robust control
f. Structured singular value