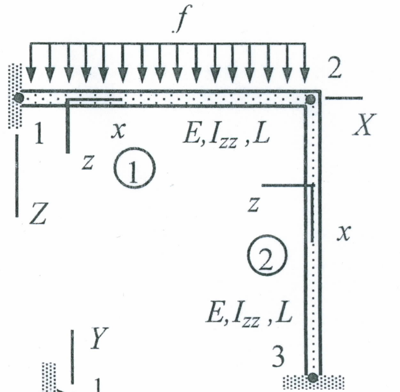
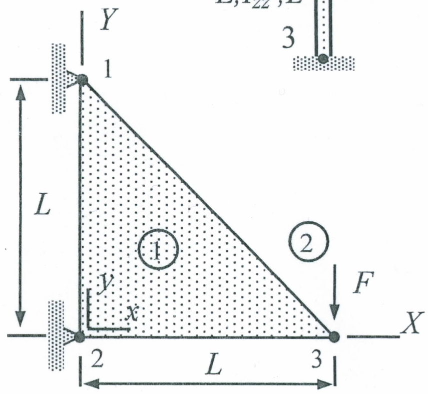
Kul-49.4100 Finite element method II, midterm 1, 25.10.2012

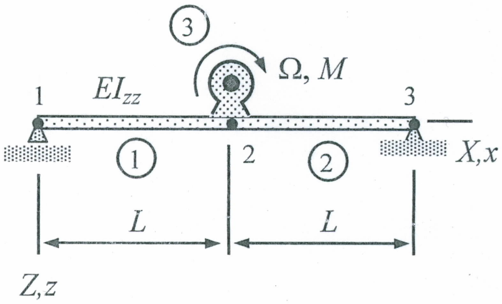
1. Determine the rotation $\theta_{Y2} = \mathrm{a}_2$ at node 2 of the structure shown. Use two Bernoulli beam elements of equal length. Assume that the beams are rigid in the axial directions. Young's modulus of the material E and the second moment of area I_{zz} are constants.



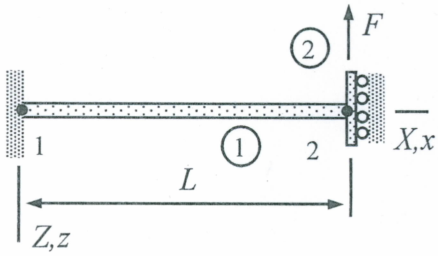
2. The thin triangular slab of figure (thickness t, plane stress) is loaded by a point force at node 3. Nodes 1 and 2 are fixed. Derive the virtual work expression $\delta W = \delta W^{\mathrm{int}} + \delta W^{\mathrm{ext}}$ in terms of $u_{X3} = \mathrm{a}_1$ and $u_{Y3} = \mathrm{a}_2$. Approximation is linear and material constants are E and $\nu = 0$.



3. The rotor of the machine shown rotates with angular speed Ω. Determine the bending stiffness EI_{zz} so that the smallest angular speed (free vibrations) of the foundation machine system coincides with Ω. The foundation is modeled as two *massless* Bernoulli beams and the machine as particle of mass M. Assume that $\theta_{Y1} = -\theta_{Y3} = \mathrm{a}_1$, $\theta_{Y2} = 0$ and $u_{Z2} = \mathrm{a}_2$.



4. The Bernoulli beam shown is subjected to force F at node 2, when $t < 0$. At $t = 0$, the force is suddenly removed and the beam starts to vibrate. Derive the ordinary differential equation and initial conditions giving the displacement $u_{Z2} = \mathrm{a}$ as function of time ($\theta_{Y2} = 0$) for $t > 0$. The beam is thin so that the rotational part of the inertia term is negligible. The geometrical quantities of the cross-section are A, I_{zz} and the material constants E and ρ.



5. Beam structure of the figure is loaded by opposite forces of magnitude p acting on nodes 2 and 3. Determine the buckling force p_{cr} of the structure using two Bernoulli beam elements. Displacements are confined to the XZ-plane. The cross-section properties of the beam A and I_{zz} and Young's modulus of the material E are constants.