Aalto University School of Electrical Engineering
Department of Communications and Networking
S-38.3143 Queueing Theory, Fall 2012

Examination
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1. Consider a single-server queue. The system is empty at time 0. New customers arrive at times 1, 4, and 5. Their service times are 7, 6, and 2, respectively. For each of the three service disciplines given below, determine the departure times of all three customers:

   (a) FB, (b) SPT, (c) SRPT.

2. Consider a renewal sequence (T_n) with finite mean $E[T] < \infty$ and Laplace transform $T^*(s) = E[e^{-sT}]$. Let $T^e(t)$ denote the corresponding elapsed lifetime process. Utilizing the theory of regenerative processes, determine the Laplace transform $E[e^{-sT^e}]$ of the steady-state elapsed lifetime distribution (as a function of the mean value $E[T]$ and the Laplace transform $T^*(s)$).

3. Let X_1 and X_2 be independent and identically distributed (IID) random variables with distribution $P\{X_i = 1\} = P\{X_i = 2\} = 1/2$ for both $i = 1, 2$. Determine the expectation $E[Y]$ for the random variable
$$Y = \sum_{i=1}^{N} X_i,$$
where N is as defined below.

   (a) Let N be an independent random variable with distribution $P\{N = 1\} = P\{N = 2\} = 1/2$.

   (c) Let $N = X_2$.

4. Consider an $M/E_2/1$-FIFO queue with $\rho < 1$. So we assume that the service times follow the Erlang distribution with two phases,
$$P\{S \le x\} = 1 - e^{-2\mu x}(1 + 2\mu x).$$
Let $X(t)$ denote the queue length at time t. In addition, let $Z(t)$ denote the phase of the customer in service (if any). If the system is empty, then $Z(t) = 0$. The pair $(X(t), Z(t))$ is a Markov process.

   (a) Draw the state transition diagram of the Markov process $(X(t), Z(t))$.

   (b) Utilizing the Pollaczek-Khinchin mean value formula, determine the mean steady-state waiting time $E[W]$ (as a function of λ and μ).

5. Consider an M/G/1-LIFO-PR queue with $\rho < 1$. Let $x > 0$. Utilizing the properties of the LIFO-PR discipline and the fact that the mean busy period is $E[B] = E[S]/(1-\rho)$, determine the mean delay of a customer whose service time is x.