Aalto University, School of Electrical Engineering **Exam 17 December 2012**
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S-88.4212 Signal Processing in Telecommunications II (4 cu)

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used.

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2-3 lines of text) the following concepts:

a) TED
b) Channel estimation
c) NDA phase estimation
d) Feedforward estimate
e) Synchronous sampling
f) Derivative matched filter

2. a) (3p) Explain the principles of the Costas loop for phase synchronization, with block diagram. Provide the iterative recursion and make connections to the Likelihood function.

b) (3p) What is the S-curve? Describe, with aid of different regions of a typical S-curve and Costas update recursions, the terms *hangup*, *phase slip*, *stable* and *unstable phase equilibriums*.

3. (7p BONUS) Consider the observed data set

$$r(k) = A + w(k), \qquad k = 0,1,\ldots, N-1 \tag{1}$$

where A is an unknown positive DC level ($A > 0$), $w(k)$ is additive white Gaussian noise *with unknown variance A*. This problem differs from the one we have seen in the lectures. Here the unknown parameter A is reflected in the mean *and* the variance. The probability density function (pdf) is

$$p(\mathbf{r};A) = \frac{1}{(2\pi A)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2A}\sum_{k=0}^{N-1}[r[k]-A]^2\right\} \tag{2}$$

a) (3p) Find the ML estimator for A in closed form.
b) (4p) Determine the Cramer-Rao bound for the estimator for A in closed form.

Hint 1: the Cramer-Rao bound for a scalar parameter is defined as the *expected value* of the negative inverse of the second derivative of the pdf with respect to the parameter in question.

Hint 2: useful variance formula $\sigma_r^2 = E\left[(r(k)-\bar{r})^2\right] = E\left[r^2(k)\right] - \left(E\left[r(k)\right]\right)^2$

(Please turn the page for more questions!)

4. (7p BONUS) Consider the carrier recovery problem in an 8-PSK system. Assume that after the first (analog) carrier modulation stage a low-frequency modulating sinusoid still remains so that the appropriate baseband signal model is

$$\tilde{x}(t) = e^{j(2\pi f t+\theta)} \sum_k a_k h_T(t - kT - \tau) \tag{3}$$

where f is the carrier frequency (error) to be estimated, carrier phase θ is unknown but the timing error τ is known. Data symbols a_k are known because a training signal is used at the beginning of transmission. The matched filter output $y(k)$ can be developed under certain assumptions (Nyquist pulse, small enough f) in the form

$$z(k) = a_k^* y(k) = e^{j[2\pi f(kT+\tau)+\theta]} + n'(k) \tag{4}$$

where $n'(k)$ is zero-mean Gaussian noise.

a) (2p) You have two samples of $z(k)$ available. Develop an estimator for the carrier frequency.

b) (3p) You have three samples of $z(k)$ available. Develop at least two estimators for the carrier frequency that utilize all the three samples and discuss their properties and possible problems.

c) (2p) You have three samples of the matched filter output $y(k)$ available. In order to save transmission bandwidth, develop an estimator that *does not require knowledge* of the transmitted data symbols a_k. The estimator should utilize all three samples.

5. (7p Bonus) Let us consider a digital transmission system with symbol rate $1/T$. The receiver has non-synchronized sampling at rate $1/T_s$. Provided the received sample values $\{z(mT_s)\}$ we would like to reconstruct the values $\{z(t_n)\}$ through interpolation, where $t_n = nT/N + \tau$ is the desired interpolation instant, N being an oversampling factor.

a) (1p) To perform the interpolation using received samples we may use FIR filters. However, the desired time instant t_n is expressed in terms of the transmitter clock. Rewrite t_n in terms of the receiver clock period T_s by properly introducing a basepoint index l_n and a fractional delay μ_n.

b) (3) Timing correction can be implemented with an FIR-type fractional delay (FD) filter whose transfer function is of the form (with causal indexing)

$$H(z) = \sum_{i=0}^{N} h_i(\mu) z^{-i} \tag{5}$$

The filter coefficients vary with the fractional delay parameter μ. Derive the Farrow structure for the (causal) interpolator whose $N+1 = 4$ filter coefficients are approximated as

$$\begin{aligned} h_0 &= 0.5\mu^2 - 0.5\mu \\ h_1 &= -0.5\mu^2 - 0.5\mu + 1 \\ h_2 &= -0.5\mu^2 + 1.5\mu \\ h_3 &= 0.5\mu^2 - 0.5\mu \end{aligned} \tag{6}$$

Produce as simple structure as possible and draw a detailed block diagram showing fixed and variable coefficient values.

(Please turn the page for more questions!)

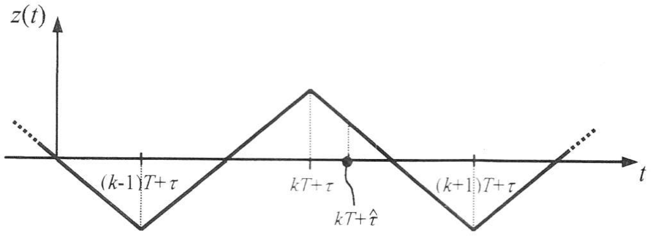

Fig. 1. Matched filter output for case of rectangular transmit pulses $h_T(t)$.

c) (3) Every Nth sample, a timing error detector (TED) produces an error signal which is proportional to the timing error $e(k)$. The timing estimate is then refined as

$$\hat{\tau}_{k+1} = \hat{\tau}_k + \gamma e(k) = \hat{\tau}_k + \gamma S(\tau - \hat{\tau}_k) + \gamma n(k) \qquad (7)$$

Assuming a high signal-to-noise ratio (SNR), Figure 1 shows the matched filter output for the case when the transmit filters $h_T(t)$ are rectangular pulses of length T. We see that for time instant k, our timing estimate is such that we sample the waveform at $t = kT + \hat{\tau}$. To refine the estimate through (7), you are presented the following error signals corresponding to four (4) different TEDs

$$e_1(k) = (a_{k-1} - a_k)\left[z(kT + T/2 + \hat{\tau}_k) - z(kT - T/2 + \hat{\tau}_k)\right]$$
$$e_2(k) = a_k z'(kT + \hat{\tau}_k)$$
$$e_3(k) = a_{k-1} z'(kT + \hat{\tau}_k)$$
$$e_4(k) = -(a_k - a_{k-1}) z(kT - T/2 + \hat{\tau}_k)$$

where z' denotes the output of the derivative matched filter.

For *each* of the error signals above, comment on whether it is suitable for timing correction. You may assume the symbols $[a_{k-1}, a_k, a_{k+1}] = [-1, 1, -1]$ known. Please *justify your answers* using the error signals, waveform of Fig. 1, and Eq. (7).