S-72.3410 Coding Methods
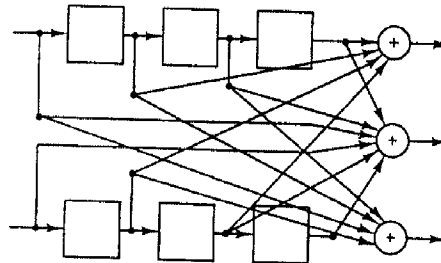

1. Consider the convolutional encoder shown above.

   (a) (3p.) Find the transfer function matrix for this encoder.

   (b) (3p.) By using this matrix, determine the codeword associated with the input sequence $x = (11, 10, 01)$.

2. Consider the ring $R_4 = GF(2)[x]/(x^4 + 1)$.

   (a) (2p.) How many elements are there in R_4? Justify your answer.

   (b) (2p.) Multiply the polynomials $a(x) = x^3 + x^2 + 1$ and $b(x) = x^2 + x$ in R_4.

   (c) (2p.) Find a non-trivial ideal in R_4. *Hint:* the smallest non-trivial ideal has only two elements; the theory of cyclic codes may be helpful here.

3. (a) (3p.) What are the possible dimensions of a binary cyclic code of length 37? What about the possible dimensions of a 16-ary cyclic code of length 37? Justify your answers.

   (b) (3p.) Find the generator polynomial of a two-error-correcting 128-ary cyclic code of length 127. Express the coefficients of the generator polynomial as powers of a primitive element of GF(128). *Hint:* vector space representations of the elements of a certain Galois field may be useful here.

4. Consider the 4-ary code C defined by the following parity-check matrix, where α is primitive in GF(4).

$$H = \begin{bmatrix} \alpha & \alpha^2 & 1 & 1 \\ \alpha^2 & \alpha & 1 & 0 \end{bmatrix}$$

   (a) (4p.) Write out the 16 code words in C.

   (b) (2p.) Is C maximum-distance separable? Justify your answer.