Buckingham Π-theorem:

If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k-r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

Criteria for the repeating variables:

1. The number of repeating variables is equal to the number of reference dimensions.
2. All the required reference dimensions must be included within the group of repeating variables.
3. Each repeating variable must be dimensionally independent of the others.

Moody chart is at the end of the exam

Colebrook formula

$$\frac{1}{\sqrt{f}} = \begin{cases} -2.0\,\log_{10}\left[\dfrac{\varepsilon/D}{3.7} + \dfrac{2.51}{\mathrm{Re}_D}\left(\dfrac{1}{\sqrt{f}}\right)\right], & \text{implicit form} \\ -1.8\,\log_{10}\left[\left(\dfrac{\varepsilon/D}{3.7}\right)^{1.11} + \dfrac{6.9}{\mathrm{Re}_D}\right], & \text{explicit form} \end{cases}$$

Material derivative

$$\frac{\mathrm{D}\varrho}{\mathrm{D}t} = \frac{\partial \varrho}{\partial t} + \vec{V}\cdot\nabla\varrho = \frac{\partial \varrho}{\partial t} + u\,\frac{\partial \varrho}{\partial x} + v\,\frac{\partial \varrho}{\partial y} + w\,\frac{\partial \varrho}{\partial z}$$

Continuity equation:

$$\frac{\partial \varrho}{\partial t} + \nabla\cdot\left(\varrho\vec{V}\right) = \frac{\mathrm{D}\varrho}{\mathrm{D}t} + \varrho\left(\nabla\cdot\vec{V}\right) = 0$$

Navier-Stokes equations: (gravity acts in negative z-direction)

$$\begin{aligned} \varrho\,\frac{\mathrm{D}u}{\mathrm{D}t} &= -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ \varrho\,\frac{\mathrm{D}v}{\mathrm{D}t} &= -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \\ \varrho\,\frac{\mathrm{D}w}{\mathrm{D}t} &= -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} - \varrho\,\mathrm{g} \end{aligned}$$

Viscous stress for a newtonian fluid: (expressed in *cartesian index notation*)

$$\tau_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) + \delta_{ij}\,\lambda\,\frac{\partial u_k}{\partial x_k}$$