Kul-34.3100 Virtausmekaniikan perusteet

Tentti

22.5.2012

Muistathan, että perustelut ovat tärkeä osa laskua ja arvostelua!

Properties of air
density: $\varrho_{\text{air}} = 1.23\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{air}} = 1.79 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2$

Properties of water
density: $\varrho_{\text{water}} = 1\,000\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{water}} = 1.12 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$

Gravitational acceleration: $\text{g} = 9.81\,\text{m/s}^2$

Equations When you use these equations, please explain what you are doing and what principle you are applying. Not all the equations may be needed.

Bernoulli equation: $p + \varrho \text{g} h + \frac{1}{2}\varrho V^2 = p_T$

Energy balance:

$$\left(p + \varrho \text{g} h + \tfrac{1}{2}\varrho V^2\right)_{\text{out}} = \left(p + \varrho \text{g} h + \tfrac{1}{2}\varrho V^2\right)_{\text{in}} + \text{work done on the CV} - \text{losses}$$

Losses: $\Delta p_{\text{friction}} = \left(f\,\dfrac{L}{D}\right)\tfrac{1}{2}\varrho V^2$ and $\Delta p_{\text{loss}} = K\,\tfrac{1}{2}\varrho V^2$

Reynolds number: $\text{Re}_l = \dfrac{\varrho V l}{\mu} = \dfrac{V l}{\nu}$

Power: $P = \Delta p\, Q$

Mass flux: $\dot{m} = \int_A \varrho \vec{V}\cdot\vec{n}\,\text{d}A$

Momentum flux: $\int_A \vec{V}\,\varrho\vec{V}\cdot\vec{n}\,\text{d}A$

Momentum balance: $\sum\vec{F} = \text{momentum flux out} - \text{momentum flux in}$

Moment-of-momentum equation:

$$\Sigma\vec{T} = \dot{m}_{\text{out}}\left(\vec{r}\times\vec{V}\right)_{\text{out}} - \dot{m}_{\text{in}}\left(\vec{r}\times\vec{V}\right)_{\text{in}}$$

$$\vec{r}\times\vec{V} = \pm r\,V_\theta$$

Euler turbomachine equation:

$$P = \dot{m}\left(\pm U\,V_\theta\right)_{\text{out}} - \dot{m}\left(\pm U\,V_\theta\right)_{\text{in}}$$

Buckingham Π-theorem:

If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k-r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

Criteria for the repeating variables:

1. The number of repeating variables is equal to the number of reference dimensions.
2. All the required reference dimensions must be included within the group of repeating variables.
3. Each repeating variable must be dimensionally independent of the others.

Moody chart is at the end of the exam

Colebrook formula

$$\frac{1}{\sqrt{f}} = \begin{cases} -2.0 \log_{10}\left[\dfrac{\varepsilon/D}{3.7} + \dfrac{2.51}{\mathrm{Re}_D}\left(\dfrac{1}{\sqrt{f}}\right)\right], & \text{implicit form} \\ -1.8 \log_{10}\left[\left(\dfrac{\varepsilon/D}{3.7}\right)^{1.11} + \dfrac{6.9}{\mathrm{Re}_D}\right], & \text{explicit form} \end{cases}$$

Material derivative

$$\frac{\mathrm{D}\varrho}{\mathrm{D}t} = \frac{\partial \varrho}{\partial t} + \vec{V}\cdot\nabla\varrho = \frac{\partial \varrho}{\partial t} + u\frac{\partial \varrho}{\partial x} + v\frac{\partial \varrho}{\partial y} + w\frac{\partial \varrho}{\partial z}$$

Continuity equation:

$$\frac{\partial \varrho}{\partial t} + \nabla\cdot\left(\varrho\vec{V}\right) = \frac{\mathrm{D}\varrho}{\mathrm{D}t} + \varrho\left(\nabla\cdot\vec{V}\right) = 0$$

Navier-Stokes equations: (gravity acts in negative z-direction)

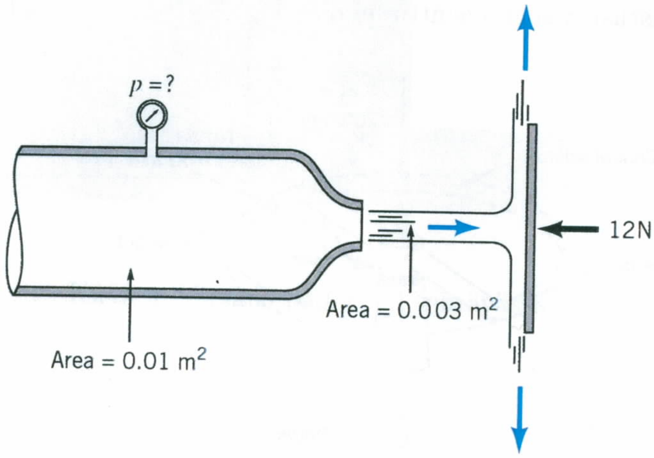
$$\begin{aligned} \varrho\frac{\mathrm{D}u}{\mathrm{D}t} &= -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ \varrho\frac{\mathrm{D}v}{\mathrm{D}t} &= -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \\ \varrho\frac{\mathrm{D}w}{\mathrm{D}t} &= -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} - \varrho g \end{aligned}$$

Viscous stress for a newtonian fluid: (expressed in *cartesian index notation*)

$$\tau_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) + \delta_{ij}\,\lambda\,\frac{\partial u_k}{\partial x_k}$$

1. [4 pt]

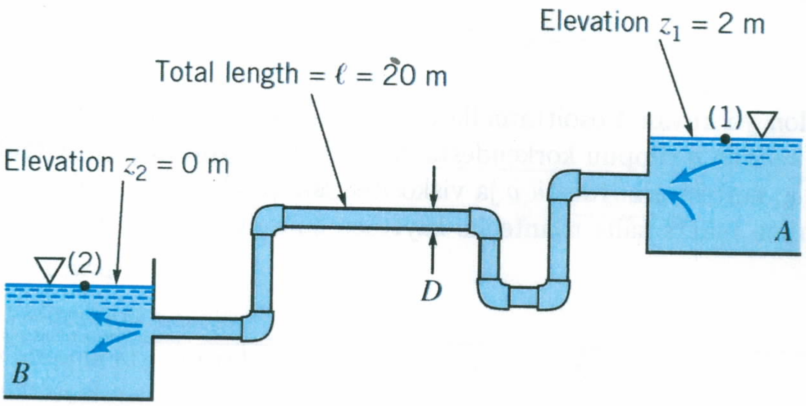
Ilma virtaa kuvan 1 mukaisen suuttimen läpi ja törmää tasolevyyn. Vaakasuora voima suuruudeltaan $F = 12\,\mathrm{N}$ tarvitaan pitämään levy paikallaan. Määritä säiliön paine. Oleta virtaus puristumattomattomaksi ja kitkattomaksi.



Kuva 1: Tehtävän 1. paineistettu suutin.

2. [6 pt]

Vesi virtaa säiliöstä A säiliöön B valurautaista putkea pitkin, jolle pinnankarheus on $\varepsilon = 0.26\,\mathrm{mm}$, pituus $l = 20\,\mathrm{m}$ ja tilavuusvirta $Q = 0.002\,\mathrm{m}^3/\mathrm{s}$ (Kuva 2). Putkistolla on teräväreunaiset lähtö- ja ulostuloaukot ja kuusi 90° mutkaa. Määritä tarvittava putken halkaisija. Vastuskertoimet ovat: $K_{\mathrm{mutka}} = 1.5$, $K_{\mathrm{ent}} = 0.5$ ja $K_{\mathrm{exit}} = 1.0$.



Kuva 2: Tehtävän 2. geometria.