Kul-34.3100 Introduction to Fluid Mechanics

First mid-term exam 07.03.2013

Remember that it is crucial to explain what you are doing and why. This strongly affects the grading.

Properties of air

density: $\varrho_{\text{air}} = 1.23\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{air}} = 1.79 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2$

Properties of water

density: $\varrho_{\text{water}} = 1\,000\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{water}} = 1.12 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$

Gravitational acceleration: $g = 9.81\,\text{m/s}^2$.

Equations When you use these equations, please explain what you are doing and what principle you are applying. Not all the equations may be needed.

Bernoulli equation: $p + \varrho\, g\, h + \frac{1}{2}\,\varrho\, V^2 = p_T$

Energy balance:

$$\left(p + \varrho\, g\, h + \tfrac{1}{2}\,\varrho\, V^2\right)_{\text{out}} = \left(p + \varrho\, g\, h + \tfrac{1}{2}\,\varrho\, V^2\right)_{\text{in}} + \text{work done on the CV} - \text{losses}$$

Losses: $\Delta p_{\text{friction}} = \left(f\,\dfrac{L}{D}\right)\,\frac{1}{2}\,\varrho\, V^2$ and $\Delta p_{\text{loss}} = K\,\frac{1}{2}\,\varrho\, V^2$

Reynolds number: $\text{Re}_L = \dfrac{\varrho\, V\, L}{\mu}$

Power: $P = \Delta p\; Q$

Mass flux: $\dot{m} = \int_A \varrho\, \vec{V} \cdot \vec{n}\, \mathrm{d}A$

Momentum flux: $\int_A \vec{V}\, \varrho\, \vec{V} \cdot \vec{n}\, \mathrm{d}A$

When velocity is constant on surface A, momentum flux: $\dot{m}\,\vec{V}$

Momentum balance: $\sum \vec{F} = \text{momentum flux out} - \text{momentum flux in}$

Moment of momentum equation:

$$\Sigma \vec{T} = \dot{m}_{\text{out}} \left(\vec{r} \times \vec{V}\right)_{\text{out}} - \dot{m}_{\text{in}} \left(\vec{r} \times \vec{V}\right)_{\text{in}}$$

$$\vec{r} \times \vec{V} = \pm r\, V_\theta$$

Euler turbomachine equation:

$$P = \dot{m}\, (\pm U\, V_\theta)_{\text{out}} - \dot{m}\, (\pm U\, V_\theta)_{\text{in}}$$

kg/m³ m² m/s

Buckingham Π-theorem:

If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k-r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

Criteria for the repeating variables:

1. The number of repeating variables is equal to the number of reference dimensions.
2. All the required reference dimensions must be included within the group of repeating variables.
3. Each repeating variable must be dimensionally independent of the others.

Moody chart and basic potential functions are at the end of the exam

Material derivative

$$\frac{\mathrm{D}\varrho}{\mathrm{D}t} = \frac{\partial \varrho}{\partial t} + \vec{V} \cdot \nabla \varrho = \frac{\partial \varrho}{\partial t} + u\,\frac{\partial \varrho}{\partial x} + v\,\frac{\partial \varrho}{\partial y} + w\,\frac{\partial \varrho}{\partial z}$$

Continuity equation:

$$\frac{\partial}{\partial t}\int_{\mathcal{V}} \varrho\,\mathrm{d}\mathcal{V} + \int_{A} \varrho \vec{V} \cdot \vec{n}\,\mathrm{d}A = 0$$

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot \left(\varrho \vec{V}\right) = \frac{\mathrm{D}\varrho}{\mathrm{D}t} + \varrho\left(\nabla \cdot \vec{V}\right) = 0$$

Navier-Stokes equations: (gravity acts in negative z-direction)

$$\varrho\,\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$\varrho\,\frac{\mathrm{D}v}{\mathrm{D}t} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$\varrho\,\frac{\mathrm{D}w}{\mathrm{D}t} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} - \varrho\,g$$

Viscous stress for a newtonian fluid: (expressed in *cartesian index notation*)

$$\tau_{ij} = \mu\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) + \delta_{ij}\,\lambda\,\frac{\partial u_k}{\partial x_k}$$

Problem 1. [6 pts.]

As a valve is opened, water flows through the diffuser shown in Fig. 1 at an increasing flowrate so that the velocity along the centerline is given by $\vec{V}(x,t) = u\hat{\imath} = V_o\,(1 - x/\ell)\,[1 - \exp(-ct)]\,\hat{\imath}$ where V_o, ℓ and c are constants. Determine the acceleration as a function of x and t. If $V_o = 10\,\mathrm{m/s}$ and $\ell = 5\,\mathrm{m}$, what *non-zero* value of c is needed to make the acceleration zero for any x at $t = 1\,\mathrm{s}$?

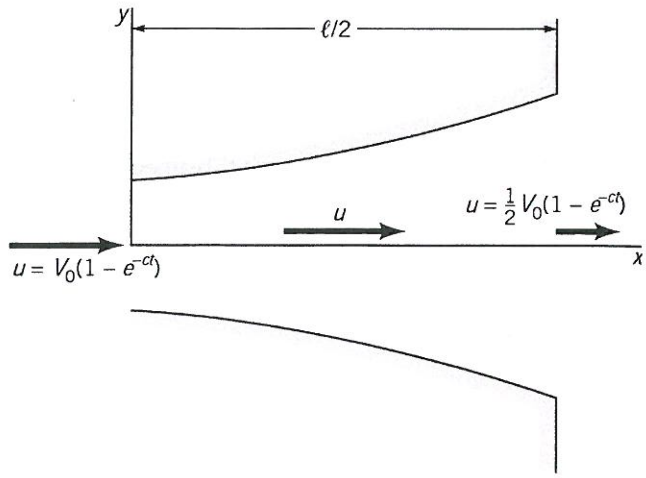


Figure 1: A diffusor of Problem 1.

Problem 2. [6 pts.]

Air flows steadily from a tank, through a hose of diameter $D = 0.03$ m, and exits to the atmosphere from a nozzle of diameter $d = 0.01$ m as shown in Fig. 2. The pressure in the tank remains constant at 3.0 kPa (gage) and the atmospheric conditions are standard temperature and pressure. Determine the flow rate and the pressure in the hose.

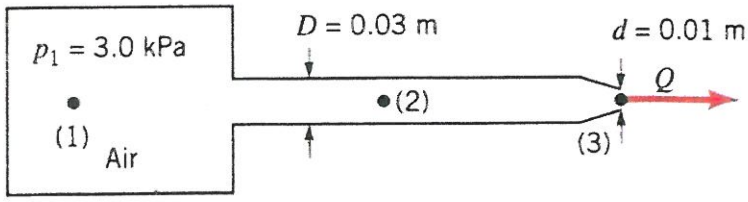


Figure 2: A geometry of Problem 2.

Problem 3. [6 pts.]

A nozzle is shown in Fig. 3, where $D_1 = 8\,\mathrm{cm}$, $D_2 = 5\,\mathrm{cm}$, and $p_2 = 1\,\mathrm{atm} \approx 101\,\mathrm{kPa}$. If $V_1 = 5\,\mathrm{m/s}$ and the manometer reading is $h = 58\,\mathrm{cm}$, where $\varrho_{\mathrm{Hg}} =$

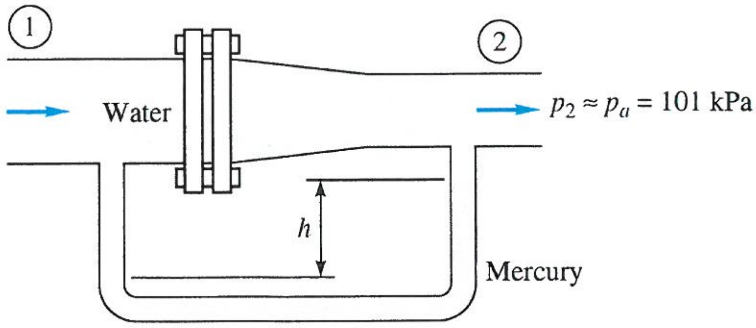


Figure 3: A nozzle of Problem 3.

$13.5 \times 10^3\,\mathrm{kg/m^3}$, estimate the *horizontal* force at the bolted connector. How much *head loss* is experienced by the flow?

Problem 4. [6 pts.]

A pump is pumping water via a pipeline from the lower pond to the upper one (Fig. 4). The pump head is 76 m ($7.46 \cdot 10^5$ Pa). Determine the pump power. Surface difference between the ponds is 60 m. The diameter of the pipe is 23 cm, and the length is 152 m. The inner surface of the pipe is smooth. The loss coefficient of a valve in the pipeline is 5 and the coefficient of the pipe bends 1.5. The outflow and the inflow loss coefficients are 1 and 0.8, respectively.

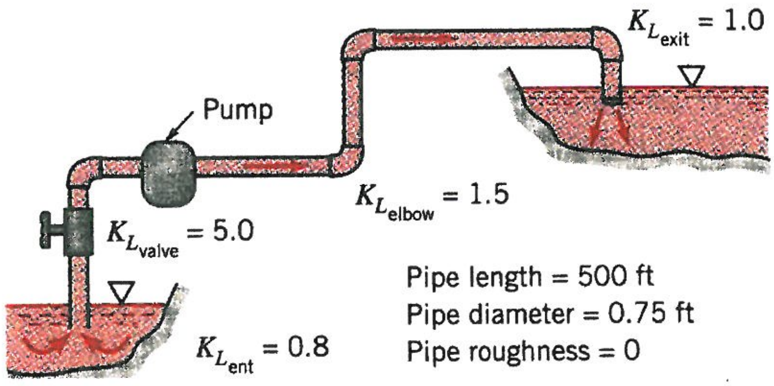
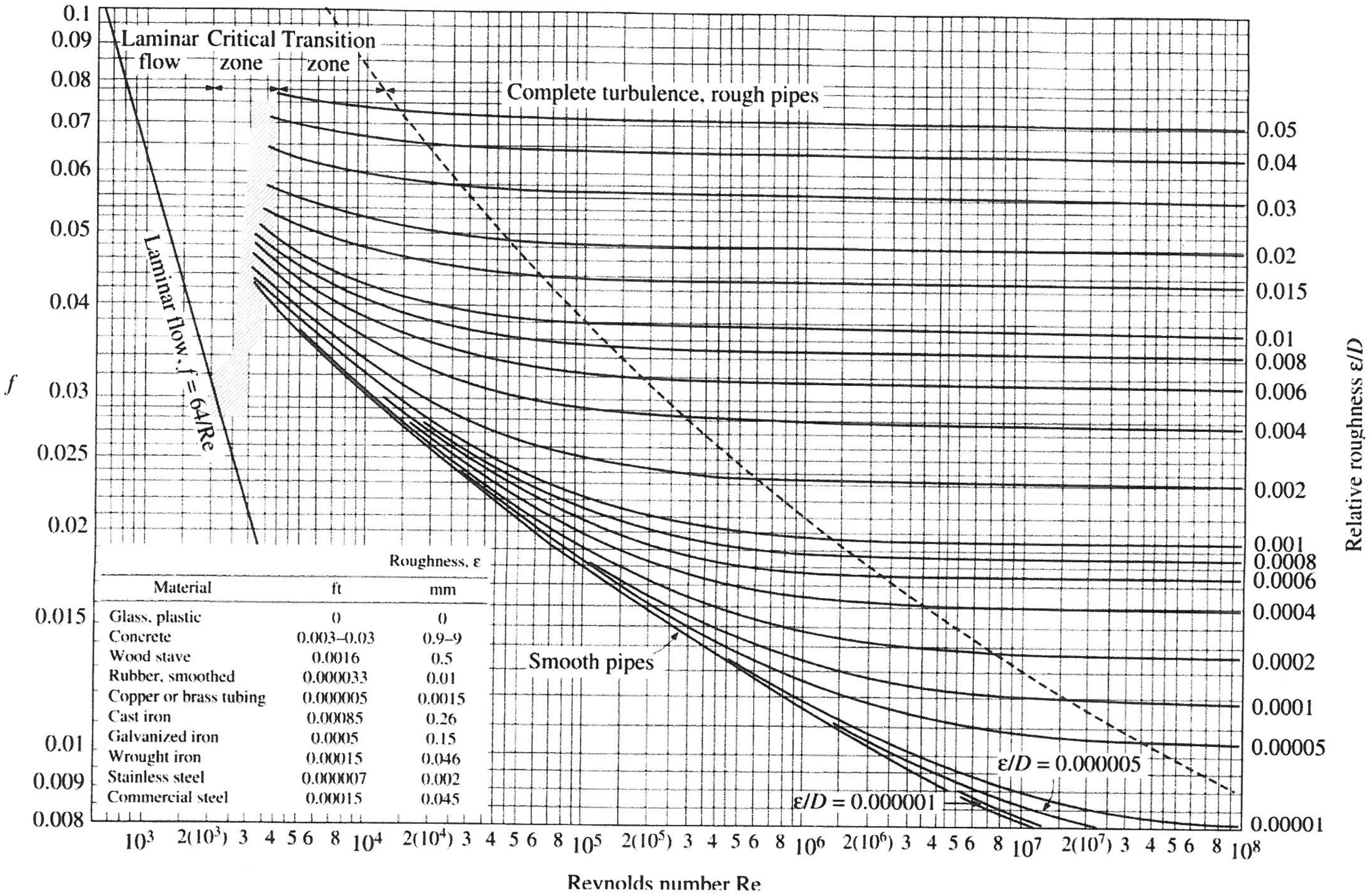


Figure 4: A pipeline between two ponds (Problem 4).



Material	Roughness, ε (ft)	Roughness, ε (mm)
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045