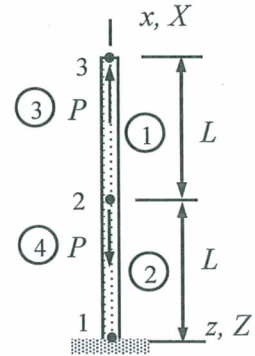
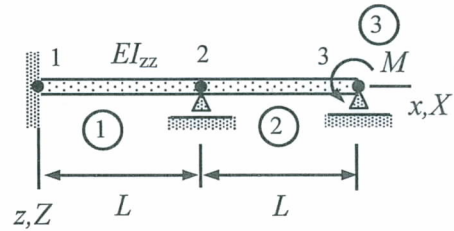


# Kul-49.3300 Finite element method I, midterm 1, 07.03.2013

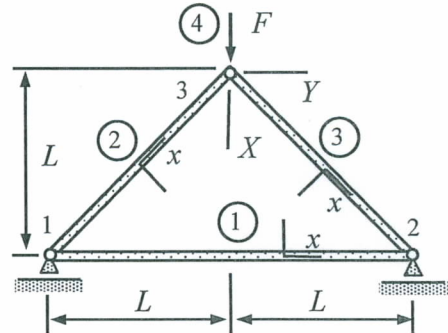
- The bar structure shown is loaded by point forces (3), (4) of equal magnitude  $P$  but opposite directions. Determine the nodal displacements  $u_{X2} = a_1$  and  $u_{X3} = a_2$ . Cross-sectional area  $A$  and Young's modulus  $E$  are constants. Use two bar elements and two force elements as indicated in the figure.



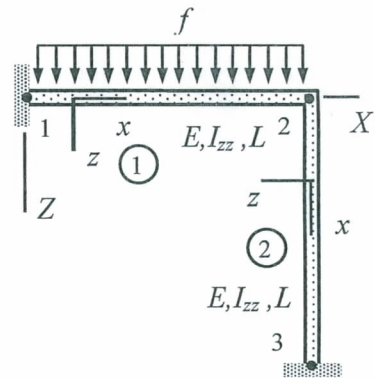
- Consider the beam of the figure loaded by moment  $M$  acting at node 3. Determine rotations  $\theta_{Y2} = a_1$  and  $\theta_{Y3} = a_2$ . Use two beam elements and one moment element. Young's modulus  $E$  and the second moment of area  $I_{zz}$  are constants.



- Determine the nodal displacements of the bar structure consisting of bar elements (1), (2) and (3) and force element (4). Assume that  $u_{Y2} = -u_{Y1} = a_1$ ,  $u_{Y3} = 0$ , and  $u_{X3} = a_2$ . Cross-sectional area  $A$  and Young's modulus  $E$  are same for all the bars.



- Determine the rotation  $\theta_{Y2} = a_1$  at node 2 of the structure. Use two Bernoulli beam elements of equal length. Assume that the beams are rigid in the axial directions. Young's modulus of the material  $E$  and the second moment of area and  $I_{zz}$  are constants.



- Consider the torsion bar (1) of the figure loaded by torque  $M$  (2) acting on the free end. Determine the rotation  $\theta_{x2} = a_1$  at the free end if  $I_{rr}$  is constant and shear modulus  $G$  varies linearly so that the values at the nodes are  $G_1$  and  $G_2$ . Start with the virtual work density  $\delta w^{\text{int}} = -\delta\phi_{,x} G I_{rr} \phi_{,x}$  and use linear approximation to rotation (a linear two-node element).

