Aalto University
Department of Information and Computer Science
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T-79.5205 Combinatorics (5 cr)
Exam Mon 20 May 2013,
9–12 a.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5205 Combinatorics 20.05.2013"
- The total number of answer sheets you are submitting for grading

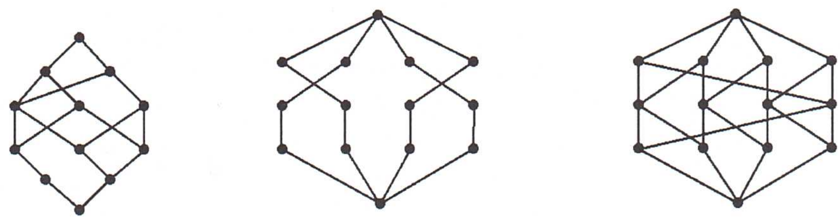
1. Give a closed-form solution or a counting recurrence as a function of n (resp. n and k) for:

   (a) The number P_n of distinct *pairings* of the set $[2n]$. (A *pairing* is a partition of a set into 2-elements subsets.) What is this number for the cases $n = 5, 6, 7$?

   (b) The number of ways we can choose a k element subset of $[n]$, $n \geq 2k-1$, such that the difference between any two (consecutive) elements is at least 2. Count these numbers for the following (n,k) pairs: (6,3), (9,3), (11,4).

2. Partially ordered sets and Lattices.

   (a) Define the notions of meet, joint, lattice, and distributive lattice. Which of the following Hasse diagrams are describing a lattices? Prove your answer.



   (b) Prove that the lattice of positive divisors of n is distributive.

   (c) Given a finite poset $(P, \leq)$, show that $(P, \leq)$ is a lattice if and only if P has a greatest element and every pair of elements has a meet.

3. Packing and Covering

   (a) What is a Steiner Triple System of order v?

   (b) Let $X = \mathbb{Z}_{13}$, and let $B_1 = \{0,1,4\}$ and $B_2 = \{0,2,8\}$. Let $\mathcal{B} = \{B_1 + z, B_2 + z : z \in X\}$. Prove that the pair $(X, \mathcal{B})$ forms a *Steiner triple system* of order 13. (Hint: Note that for any nonzero $z \in X$, there is a unique way to write z as $z = x - y$ with $x, y \in B_i$)

4. Extremal set Theory

   (a) What is a *sunflower* with k *petals*? Prove that if $\mathcal{F}$ is an s-uniform set family with $|\mathcal{F}| > s!(k-1)^s$, then $\mathcal{F}$ contains a sunflower with k petals (Sunflower Lemma).

   (b) Show that any finite graph contains two vertices lying on the same number of edges.

Grading: Each problem 12p, total 48p.