Aalto University, School of Electrical Engineering — **Exam May 20 2013**

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S-88.2311 Signal Processing in Telecommunications I (5 cp)

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:

a) Generalized matched filter
b) DFE
c) Precursor ISI
d) MLSD
e) Echo cancellation
f) ZF equalizer

2. MSE gradient algorithm for an FIR adaptive equalizer (6p):

Let us consider a discrete-time model for a communication system in a linear channel (sampled at the symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output $y(k)$ can be expressed as

$$y(k) = \boldsymbol{h}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{r}(k)$$

where $\mathbf{h}_{\mathrm{R}}$ and $\mathbf{r}$ are N-dimensional column vectors. Draw the receiver block diagram and derive the MSE gradient (MSEG) adaptive algorithm to update the equalizer coefficients. What is the optimal equalizer solution and in what conditions does the adaptive algorithm reach it? Discuss the convergence issues too.

3. Nyquist criterion (7p total):

Let us derive pulse waveforms which meet the Nyquist criterion:

a) (1p) Assume an ideal baseband communication system of data rate $1/T$ in an AWGN channel. Define the *pulse spectrum* and derive the *continuous-time pulse waveform* via inverse Fourier transform. No filtering assumed in the receiver, just symbol-rate sampling.

b) (3p) The same as a) except now we assume excess bandwidth α=0.5 and that the spectrum is *piecewise constant*. Show also that the spectrum fulfills the Nyquist criterion (e.g., graphically).

c) (3p) The same as b) except now we want to design transmit and receive filters that form a *matched-filter pair* and whose *convolution meets the Nyquist criterion*. Assume that the *spectrum of the convolution is piecewise constant.*

4. Viterbi (7p):

Considering transmitting bits x_k (zeroes '0' and ones '1') over a channel with additive white Gaussian noise. Assume that $x_k = 0$ for ~~$k < 3$~~ and $k \geq 3$. The sequence is $r(0) = 0.6$, $r(1) = 0.9$, $r(2) = 1.3$, and $r(3) = 0.3$. $k<0$

a) (1p) Find the ML decision sequence $\hat{x}_k$ assuming that the additive noise is the only degradation (no ISI) and that x_k are i.i.d.

b) (6p) Suppose you are told that the ISI channel $h(k) = \delta_k + 0.5\delta_{k-1}$ is being used. Model the system as a shift register process and draw the trellis. Label it with the input/output pairs (x_k, q_k). What is the ML detection of the incoming bit sequence?

5. Channel capacity (7p):

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where '*' denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

a) (4p) *Solve for the optimum transmit power spectrum* $S_{x,opt}(f)$ that maximizes the channel capacity C_{CH} when the total transmit power P_x is limited to 4/3 W (≈ 1,333 W). Also, *provide the value of the associated water-filling level L.* To help you solve the problem, the PSD of the noise $S_n(f)$ (in W/Hz) and the magnitude squared of the channel transfer function $|C(f)|^2$ are given by

$$S_n(f) = \begin{cases} 1, & |f| \leq 1\text{ Hz} \\ |f|, & 1\text{ Hz} < |f| \end{cases} \qquad |C(f)|^2 = 1/|f|^2$$

b) (3p) Determine the channel capacity given the optimized transmit power spectrum in a).

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2$$

whenever resulting $S_{x,opt}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df$$

The optimal capacity is then obtained by integration: $C_{CH} = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_{x,opt}(f)|C(f)|^2}{S_n(f)}\right) df$