

Midterm Exam 2 17.5.2013

1. (a) Explain why bosons can form the Bose-Einstein condensate but fermions can not.
(b) Explain the connection between the random walk and the diffusion equation.
(c) In the 2D Ising Model there is a finite critical temperature $T_C \neq 0$ at which system undergoes the phase transition. Why there is no such phase transition in the 1D Ising Model?
2. Consider the statistics of a non-interacting gas of particles on energy states E_ℓ . On any given state, there can be up to p particles, i.e. the allowed occupation numbers are $n_\ell = 0, 1, \dots, p$. The total energy of the system is then $E = \sum_\ell n_\ell E_\ell$. (These hypothetical particles are called anyons.)
 - (a) Calculate the grand canonical partition function.
 - (b) Calculate the average occupation number $\langle n_\ell \rangle$.
 - (c) Consider the limits $p = 1$ and $p \rightarrow \infty$. What is $\langle n_\ell \rangle$ in these limits, and what are the corresponding statistics?

3. Consider a system of two interacting spins $s_{1,2} = \pm 1$, subjected to an external magnetic field B . Its Hamiltonian is

$$\mathcal{H} = Js_1s_2 - \mu_B B(s_1 + s_2), \quad (1)$$

where J and μ_B are parameters.

- (a) Find the Helmholtz free energy and the average magnetic moment $m = \mu_B \langle s_1 + s_2 \rangle$, for a given temperature and magnetic field.
 - (b) Find the linear magnetic susceptibility $\xi = \left. \frac{\partial m}{\partial B} \right|_{B \rightarrow 0}$ as a function of temperature.
4. Consider a 1D random walker that moves one step to the right with probability p_1 , two steps to the right with probability p_2 and one step to the left with probability q on an infinite lattice with lattice spacing of L (see Fig. ??).
 - (a) What is the condition that q , p_1 and p_2 must satisfy? (1 p.)
 - (b) What is the condition that q , p_1 and p_2 must satisfy for there to be no particle drift (i.e. $\langle \Delta x \rangle = 0$) after N steps ($N \rightarrow \infty$)? (2 p.)

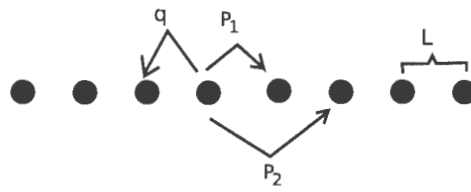


Figure 1: Random walk of question 4.

- (c) Calculate the tracer diffusion coefficient when there is no drift. (3 p.)

Recall: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$