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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Wednesday May 22nd, 2013 at 9:00–12:00

Write on every answer sheet:
Name, degree programme, student number
The text: "T-79 1001 Introduction to Theoretical Computer Science T 8.3.2013"
- The total number of answer sheets submitted for grading

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1 Finite state automata ja regular expressions.

(a) Show that the language $\{w \in \{a,b\}^* \mid \text{the number of } b\text{s in } w \text{ is divisible by } 3\}$ is regular by describing it as a regular expression. *5p.*

(b) Show that the language $\{w \in \{a,b,c\}^* \mid w \text{ contains the substring } bac \text{ or ends in the substring } bc\}$ is regular by describing it as a (possibly non-deterministic) finite state automaton. *5p.*

(c) Design the deterministic finite state automaton with the minimal number of states that accepts the language described by the regular expression $bb(cab \cup ab)^*$ *5p.*

2. Consider the language

$$L = \{a^n(ca)^m b^{n+1} \mid m \geq 0 \text{ and } n \geq 0\}$$

over the alphabet $\{a,b,c\}$

(a) Show that L is not regular. *7p.*

(b) Design a context free grammar that produces L. *6p.*

(c) Give parse trees for the strings *cacab* and *aacabbb* in your grammar. *2p.*

3. Design a Turing machine that decides the language

$$L = \{wcw \mid w \in \{a,b\}^*\}$$

over the alphabet $\{a,b,c\}$ If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally.

Give the computations of your machine with the inputs *aca* and *bacab*. *15p.*

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language.
Is the language $L_{\text{primeprod}} = \{x \in \{0,1,\ldots,9\}^* \mid x \text{ is a product of two prime numbers}\}$ recursive? Justify your answer. (E.g. 15 belongs to the language as $15 = 3 \times 5$ but 16 is not in the language.) *5p.*

(b) Prove the following claim either correct or incorrect: If L_1 is a context-free language and L_2 is a recursive language, then $L_1 \cap L_2$ is a context-free language. *5p.*

(c) Given a language L over an alphabet Σ, let $L^R = \{w^R \mid w \in L\}$ be the language obtained by reversing each string in L. Here w^R is the reverse of w (for example, $(gnat)^R = tang$). Prove the following claim either correct or incorrect: if L is a recursive language, then L^R is also a recursive language. *5p.*

Total 60p.