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**T-79.4101 Discrete Models and Search (5 cr)**
**Exam May 30, 2013**

Write down on each answer sheet:
- Your name, degree program, and student number
- The text: "T-79.4101 Discrete Models and Search 30.5.2013"
- The total number of answer sheets you are submitting for grading

*Note:* You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.

1. Which of the following claims are true and which are false? *(10 points)*
   For each claim, justify your answer using 1-3 sentences.

   (a) Local search is typically useful for proving the non-existence of solutions.

   (b) An admissible heuristic is always consistent (i.e., monotonous).

   (c) The following two CSPs are equivalent wrt. $\{x_1, x_2\}$: $\langle C_1(x_1,x_2,x_3); x_1 \in D, x_2 \in D, x_3 \in D\rangle$, and $\langle C_2(x_1,x_2); x_1 \in D, x_2 \in D\rangle$, where $D = \{1,2,3\}$, $C_1 = \{(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)\}$ and $C_2 = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$.

   (d) The following CSP is hyper-arc consistent: $\langle C_1(x,y,z), C_2(x,z); x \in \{1\}, y \in \{1,2\}, z \in \{1,2\}\rangle$, where $C_1 = \{(1,1,2),(1,2,3),(2,1,3)\}$ and $C_2 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$.

2. (a) Encode the following requirement as a linear constraint. Explain your encoding. *(5 points)*
   "*If job 1 takes place before job 2, then job 1 must end at least two time units before job 2 starts.*"

   (b) Consider the following integer program: *(5 points)*

$$
\begin{aligned}
\max\ & 2x_1 - x_2 \\
-x_1 + 3x_2 &\geq -3 \\
x_1 + 3x_2 &\leq 6 \\
x_1 &\geq 0 \\
x_1, x_2 &\quad \text{are integers}
\end{aligned}
$$

   What is the linear relaxation of this program? Transform the linear relaxation to the Simplex tableau form and give a basic feasible solution for the relaxation. Is the solution you gave optimal? Justify your answer based on the Simplex tableau.

3. Simulate DPLL (draw a DPLL search tree) on the input CNF formula consisting of the clauses *(10 points)*
   $(x_1 \vee x_2)$, $(\neg x_1 \vee x_2)$, $(\neg x_1 \vee \neg x_2)$, $(x_1 \vee \neg x_3 \vee \neg x_4)$, $(x_1 \vee x_3 \vee \neg x_4)$, $(\neg x_2 \vee \neg x_3 \vee x_4)$, $(\neg x_2 \vee x_3 \vee x_4)$.
   Use any branching heuristic you want. Clearly denote which variable assignments are made by branching and which by unit propagation in the search tree. Is the CNF formula satisfiable?

4. Consider the following optimization problem MAX CUT. *(10 points)*

   INSTANCE: A graph $G = (V,E)$ and a function $c$ giving each edge $e \in E$ an integer capacity $c(e)$.
   TASK: Find a maximum-size cut in $G$.

   A *cut* in a graph $G = (V,E)$ is a partition of the set of nodes $V$ into two nonempty sets $S$ and $V \setminus S$. The size of a cut is the sum of the capacities of the edges between $S$ and $V \setminus S$.

   Describe some simple local search heuristic for MAX CUT. Explain the following in detail: (a) what are the candidate solutions considered by your method and what is their neighborhood relation, (b) how does one choose the next solution for consideration from the neighborhood of a given candidate solution, and (c) how does one choose the initial candidate solution for the computation.