T-79.5207 Advanced Course in Algorithms (5 cr) P

Exam, 30 May 2013, 9–12 a.m.

Write down on each answer sheet:
– Your name, degree programme, and student number
– The text: "T-79.5207 Advanced Course in Algorithms 30.5.2013"
– The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. *Linear programming and approximation algorithms.*
   (a) Formulate the SET COVER problem as an integer programming problem.
   (b) Formulate the dual of the SET COVER integer program and propose a combinatorial interpretation of this dual IP.
   (c) Propose some reasonable approximation algorithm for this dual IP.

2. *Exact and parameterized algorithms.*
   (a) What is meant by problem parameterization and fixed-parameter tractability?
   (b) Show that the VERTEX COVER problem below is fixed-parameter tractable with respect to the parameter k. Carefully justify your answer.
   Hint: One possible way to approach the problem is to reduce out isolated vertices and vertices of degree more than k.

   VERTEX COVER
   Input: An undirected graph G with n vertices, an integer $k \geq 0$.
   Question: Is there a subset $S \subseteq V(G)$ of vertices with $|S| \leq k$ such that every edge of G has at least one end-vertex in S?

3. *Time–space tradeoffs.* Let us study a variant of the SUBSET SUM problem.

   k-SUM
   Input: Integers $a_1, a_2, \ldots, a_n, t, k \geq 0$.
   Question: Is there a subset $S \subseteq \{1, 2, \ldots, n\}$ such that $|S| = k$ and $\sum_{j \in S} a_j = t$?

   Suppose furthermore that $M = t + a_1 + a_2 + \ldots + a_n \geq 2$ and that k is even. Carefully justifying your answers, give algorithms that solve the k-SUM problem in worst-case
   (a) time $O(n^{k+1} \log M)$ and space $O(n \log M)$, and
   (b) time $O(n^{k/2+1} \log M)$ and space $O(n^{k/2+1} \log M)$.

4. *Randomised and stochastic algorithms.* Consider a lone rook (Finnish "torni") making random moves on an $n \times n$ chessboard, meaning that at each move, the rook chooses one of its permissible next-state squares uniformly at random. Show that for $n \geq 3$ the Markov chain defined by these moves is regular, and determine its stationary distribution. Calculate some upper bound on the mixing time of the chain.

Grading: Each problem 12p, total 48p.

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