S-72.2505 Digital Transmission Methods

Exam 20. 5. 2011

All five tasks are evaluated and taken into account in the grading. The exam can be written in Finnish, Swedish or English.

This is a closed book exam

1. Answer shortly the following questions.

   a) What is pulse shaping?

   b) Why is pulse shaping needed?

   c) Why is the sinc pulse not good for pulse shaping?

   d) What characteristics of Root Raised Cosine (RRC) pulses make them more attractive than sinc pulses?

   e) What tradeoff is controlled by the roll-off parameter in RRC pulse shaping?

2. The error probability of differential BPSK in the Additional White Gaussian Noise channel is $P_b(\gamma) = \frac{1}{2}e^{-\gamma}$, where γ is the instantaneous Signal-to-Noise Ratio (SNR).
   a) Derive the expression for average bit error probability of differential BSPK in a Rayleigh fading channel without diversity.
   b) Derive the expression for average bit error probability of differential BSPK in a Rayleigh fading channel with selection combining of two equally strong diversity branches.
   c) Calculate the **diversity gain** in dB (= the reduction in the average required SNR) for differential BPSK with two branch selection diversity at the bit error probability values 10^{-2} and 10^{-4}.
   Hint: If there are M diversity branches and $\bar{\gamma}$ is the SNR per branch, the CDF of post-combining SNR γ_c with selection diversity is $P(\gamma_c < \gamma) = \left(1 - e^{-\gamma/\bar{\gamma}}\right)^M$.

3. Channel characterization. The Mobile Station is moving with a velocity of $v = 3$ km/h, and communicating on a carrier frequency of $f_c = 2$GHz. The symbol duration is $T = 10\mu$s and the bandwidth is 100 kHz. The channel power delay consists of three channel taps

Tap i	1	2	3
Delay τ_i [μs]	0	0.2	0.5
Power P_i [dB]	-3	0	-4

   a) What is the maximum Doppler shift and the Doppler spread of the channel? Estimate the coherence time. Is the signal at the Mobile station rapidly or slowly fading?
   Hint: The speed of light is $c = 3 \cdot 10^8$m/s
   b) What are the maximum and mean excess delays of the channel? Estimate the coherence bandwidth. Is the fading frequency selective or frequency flat?

4. The Fourier transform of the unit step function at time t', $U(t-t') = \begin{cases} 1 & \text{if } t \geq t' \\ 0 & \text{if } t \leq t' \end{cases}$ is

$$U(f) = \mathrm{e}^{2\pi \mathrm{j} f t'} \left(\frac{1}{2\pi \mathrm{j} f} + \pi \delta(f) \right) .$$

Calculate the frequency response, i.e. the Fourier transform of the rectangular pulse $\mathrm{rect}(t) = U(t+\frac{T}{2}) - U(t-\frac{T}{2})$, and the power spectrum density. What can we conclude from this on the usability of the rectangular pulse for transmission in a bandlimited system?

5. The four-subcarrier DFT matrix is

$$\mathbf{F} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -\mathrm{j} & -1 & \mathrm{j} \\ 1 & 1 & 1 & 1 \\ 1 & \mathrm{j} & -1 & -\mathrm{j} \end{bmatrix} .$$

The channel is a tapped delay line with three taps, $\mathbf{h} = \begin{bmatrix} 1 & 5/6 & 1/6 & 0 \end{bmatrix}^{\mathrm{T}}$, and the transmitted SNR is $\gamma = 10$. We assume an OFDM tranmsission with a cyclic prefix of at least 3 samples, so that all ISI is removed. What is the SNR experienced at the four subcarriers? What can we say about the frequency selectivity of the channel, and the usability of the subcarriers for communication?

Hint: We are interested in the frequency domain channel, given in this case by a discrete Fourier transform.