Aalto University School of Science, Department of Information and Computer Science

T-79.3001 Logic in Computer Science: Foundations

Examination, August 12, 2013

Please note the following: your answers will be graded only if you have completed the three obligatory home assignments before the exam!

Assignment 1 $(7+3p)$

(a) Two combinatorial circuits both with a single output gate have been represented as propositional formulas $F_1(x_1,\ldots,x_n)$ and $F_2(x_1,\ldots,x_n)$, with the input gates represented as propositional variables $x_1,\ldots,x_n$.

Construct a formula for testing whether both circuits compute the same Boolean function.

What do you do with this formula to actually perform the test? See whether it is satisfiable, or valid, or something else? Justify your answer.

(b) Assume you have a procedure $V(\phi)$ for testing the *validity* of an arbitrary formula ϕ in the predicate logic. How could you use this procedure for testing the *satisfiability* of an arbitrary predicate logic formula ψ? And *logical consequence* $\phi_1 \models \phi_2$?

Assignment 2 $(10p)$ Prove the following claims using semantic tableaux:

(a) $\models (A \to B \vee D) \wedge (C \to D) \to (((A \to B) \to C) \to D)$

(b) $\{\forall x(P(x) \to R(x)), \forall x(\neg Q(x) \to \neg R(x))\} \models \forall x(P(x) \to Q(x))$

Tableau proofs must contain all intermediate steps !!!

Assignment 3 $(10p)$ Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg\forall x\exists y(\exists z Q(y,z) \to \exists v Q(x,v)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 $(10p)$

Formalize the following sentences in the predicate logic.

All students are smart.
Some criminals are not smart.
Everybody who has a student cap is a student.
____
Some criminals don't have a student cap.

Show that the last sentence is a logical consequence of the first three sentences by using semantic tableaux or resolution.

Assignment 5 $(10p)$

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=0-x; z=y; while(!(z==0)) {z=z-1; v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==y-x}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.