Mat-2.3140 Linear Programming **Toppila/Kivistö**
Final Exam **02.03.2013**

Answer all four problems, which are graded on a scale 0-6 points.

Problem 1

Are the following statements true of false? Why?

(a) Moving to a basic direction improves the objective value.

(b) At the optimum of a maximum flow problem there are no augmenting paths.

(c) The dual is feasible if and only if the primal is feasible.

(d) The simplex algorithm is polynomial time if there are no degenerate vertices.

Problem 2

Solve using the big-M–method the problem

$$\begin{aligned} \min\ & x_1 - 2x_2 + x_4 \\ \text{s.t.}\ & -x_1 + x_2 = 1 \\ & x_2 - 2x_3 + 3x_4 = 10 \\ & x_1 + x_3 + 4x_4 = 4 \\ & x_1, x_2, x_3, x_4 \geq 0\ . \end{aligned}$$

Problem 3

(a) Out of five alternatives one wishes to select as many as possible under the following constraints:

- Alternative 1 cannot be selected unless alternative 2 or 3 have been selected.
- At least 1 but at most 2 of alternatives 2-5 must be selected.
- Alternative 5 can be selected only if alternative 1 have not been selected, unless alternatives 3 and 4 have been selected.

Formulate the problem as a linear integer programming problem.

(b) Let the matrix $A \in \mathbb{R}^{n \times m}$, the vectors $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^m, c \geq 0$ be given and the decision variable be $x \in \mathbb{R}^m$. Formulate as a (continuous) linear programming problem

$$\min \quad \sum_{j=1}^{m} c_j |x_j| \ \text{ s.t. } Ax \geq b\ .$$

Problem 4

A project has phases a-k with varying durations. All phases cannot be started at once, because there are precedence requirements between the phases. These requirements and the durations are in the table below.

(a) Formulate a network problem that solves the minimal duration of the entire project. (2 p)

(b) On the critical path of the project are phases b,d,e, and h. Based on this, find the optimal primal and dual solution of part (a). (4 p)

Phase	Duration	Precedence requirements
Start		None
a	2	Start
b	3	Start
c	2	Start
d	5	b
e	2	a,d
f	4	b
g	5	c
h	1	e,f,g
i	2	c
j	3	b
k	1	i,j
Finish		h,k