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Department of Information and Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Thursday October 24th, 2013, 9:00–12:00

Ensure that every answer sheet contains:
- Name, degree programme, student number
- Name of the course "T-79.1001 Introduction to Theoretical Computer Science T" and the date "Oct 24, 2013"
- The total number of answer sheets submitted for grading

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1. Show that the following languages are regular by describing each of them as a regular expression or as a finite state automaton:
   (a) $\{w \in \{a,b\}^* \mid w$ starts or ends with the substring $abba\}$ *5p.*
   (b) $\{w \in \{a,b\}^* \mid$ the number of bs in w is even$\}$ *5p.*
   (c) $\{w \in \{a,b,c\}^* \mid w$ does not contain the substring aa or the substring $ac\}$ *5p.*

2. Consider the language $L = \{ucv \mid u, v \in \{a,b\}^* \text{ and } |v| \geq 2|u|\}$ over the alphabet $\{a,b,c\}$.
   (a) Show that L is not regular. *6p.*
   (b) Design a context free grammar that produces L. *5p.*
   (c) Give parse trees for the strings $acaba$ and $bacabbb$ in your grammar. *2p.*
   (d) Is your grammar in Chomsky normal form? If not, give one normal form requirement that is violated in your grammar. *2p.*

3. Design a Turing machine that recognises the language

   $$L = \{w \in \{a,b,c\}^* \mid w \text{ contains at least as many } b\text{s as } a\text{s}\}.$$

   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally.

   Give the computation of your machine with the inputs ba and cac. *15p.*

4. Let L_1 and L_2 be languages over an alphabet Σ.
   (a) Prove that if the language L_1 is regular and L_2 is context-free, then the language $L = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$ is also context-free. *5p.*
   (b) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language.
   Is the language $L_{\text{primeprod}} = \{x \in \{0,1,...,9\}^* \mid x \text{ is a product of two prime numbers}\}$ recursive or recursively enumerable? Justify your answer briefly. (E.g. 15 belongs to the language as $15 = 3 \times 5$ but 16 is not in the language.) *5p.*
   (c) Show that if the language L_1 is recursively enumerable and L_2 is recursive, then the language $L = L_1 \cap L_2$ is recursively enumerable. *5p.*

Total 60p.

DMN.