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Department of Information and Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Wednesday December 18th, 2013, 14:00–17:00

Ensure that every answer sheet contains:
- Your name, degree programme, student number
- Course name "T-79.1001 Introduction to Theoretical Computer Science T" and the date "Dec 18, 2013"
- The total number of answer sheets submitted for grading

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1. (a) Give a deterministic finite state automaton for the language
   $L = \{w \in \{a,b\}^* \mid w \text{ has even length and an odd number of } a\text{'s}\}$ *5p.*

   (b) Give a regular expression generating the language
   $L = \{w \in \{a,b\}^* \mid w \text{ contains at least two } a\text{'s and at most one } b\}$ *5p.*

   (c) Give a regular expression generating the language
   $L = \{w \in \{a,b\}^* \mid w \text{ does not contain the substring } aba\}$
   (Hint: you may want to first design a deterministic automaton for L) *5p.*

2. Consider the language $L = \{(ab)^k c^n a^m \mid k, n, m \geq 0 \text{ and } m = k + n\}$ over the alphabet $\{a, b, c\}$.

   (a) Show that L is not regular. *6p.*

   (b) Design a context free grammar that produces L. *5p.*

   (c) Give a parse tree for the string *abccaaa* and a derivation for the string *ababaa* in your grammar. *2p.*

   (d) Design a pushdown automaton that recognizes the language. Is your automaton deterministic? *5p.*

3. Design a Turing machine that recognises the language

$$L = \{w \in \{a,b,c\}^* \mid w \text{ contains equally many } a\text{'s, } b\text{'s and } c\text{'s}\}.$$

   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally.

   Give the computation of your machine with the inputs *aacbbc* (that is in L) and *babcc* (that is not in L). *12p.*

Continued on the other side

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language. Is the language

$$L_{\text{perfect cube}} = \{x \in \{0, 1, ..., 9\}^* \mid x = y^3 \text{ for some positive integer } y\}$$

recursive? Justify your answer briefly. (E.g., 1331 belongs to the language as $1331 = 11^3$ but 1400 is not in the language.) *5p.*

(b) Let L_1 and L_2 be languages over an alphabet Σ. Prove the following claim either correct or incorrect: if L_1 is a recursively enumerable language and L_2 is a regular language, then $L = L_1 \cap L_2$ is a context-free language. *5p.*

(c) Prove that the following problem is undecidable: given an arbitrary Turing machine, does it accept the string 1010? That is, prove that the language $L_{1010} = \{c \in \{0, 1\}^* \mid \{1010\} \subseteq L(M_c)\}$ is not recursive, where M_c denotes the Turing machine encoded by the string c. If you use "Rice's theorem", define it as well. *5p.*

Total 60p.