Aalto University
Systems Analysis Laboratory

Mat-2.3139 Nonlinear Programming Matteo Brunelli *20.12.2013*

Here there are 5 questions, each of which gives you maximum 2 points. Some questions are divided into smaller parts, for example 4a) will give you maximum one point and 4b) maximum another point. Good luck!

1. Define the following concepts analytically *and* with a small drawing: a) affine and quasi-convex functions b) conjugate directions c) convex cone d) feasible direction e) convex hull.

2. (a) Consider a nonempty closed convex set $S \subset \mathbb{R}^n$ and a point $\mathbf{y} \notin S$. Prove that there exists a point $\overline{\mathbf{x}} \in S$ with minimum distance from $\mathbf{y}$.

   (b) Prove the weak duality theorem.

3. Consider the optimization problem

$$\begin{aligned} \underset{(x_1,x_2)}{\text{minimize}} \quad & x_1^2 + x_2^2 - 14x_1 - 6x_2 - 7 \\ \text{subject to} \quad & x_1 + x_2 \leq 2 \\ & x_1 + 2x_2 \leq 3\,. \end{aligned} \tag{1}$$

   Write down the KKT conditions and use them to algebraically find the optimal solution without using the graphical representation of the problem.

4. (a) A diagonally dominant matrix is a matrix with $|a_{ii}| \geq \sum_{j \neq i} |a_{ij}|$ for all i. In the course we saw that

   - *If* **A** is diagonally dominant, symmetric and has real positive diagonal entries, *then* **A** is positive semidefinite.

   Is it true that

   - *If* **A** is positive semidefinite, symmetric, and has real positive diagonal entries, *then* **A** is diagonally dominant?

   Prove your conclusion.

   (b) Explain Armijo's rule and in what problems it can be profitably applied.

5. Present:

   (a) and justify the update rule in Newton's method for multidimensional unconstrained optimization

   (b) the concept of penalty function, its use in constrained optimization and possible issues with this method.