AS-74. 3123 Model-based control systems
Exam 12.12.2013

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the mandatory project work have been accepted. The (voluntary) homework problems constitute one problem in the exam (the 5th problem).

4 problems.

1.   Explain briefly the following concepts

- Principle of Optimality
- Dynamic programming
- Waterbed effect
- Robust stability
- Small gain theorem
- Internal model control

2.a. Draw a schema of the "two-degrees-of-freedom" control configuration. Define the concepts *sensitivity function* and *complementary sensitivity function* for it.

2.b. Prove the following relationships (in the formulas S and T are the sensitivity and complementary sensitivity functions, G is the process transfer function matrix and F_y is the compensator; the dimensions are assumed to be appropriate). Note: if you want to use the "push-through rule" directly, you must formulate and prove it first.

$$S(j\omega)+T(j\omega) = I$$
$$G(j\omega)(I + F_y(j\omega)G(j\omega))^{-1} = (I + G(j\omega)F_y(j\omega))^{-1}G(j\omega)$$

2.c. Consider the SISO-case. Determine the regions in the complex plane where $|S|$ is smaller than 1, equal to 1, and larger than 1. How can the result be explained in view of control performance?

3. Consider a SISO-process with the transfer function

$$G(s) = \frac{s + \dfrac{1}{T_1}}{\left(s + \dfrac{1}{T_2}\right)\left(s + \dfrac{1}{T_3}\right)}, \quad T_1 < 0, \quad T_2 < 0, \quad T_3 > 0$$

Explain, what kind of fundamental limitations on control performance can be stated for this system.

4. Consider the system

$$\dot{x}(t) = u(t), \quad x(0) = x_0$$

Calculate the control law, which minimizes the criterion

$$J = \frac{1}{2}\int_0^\infty (x^2(\tau) + u^2(\tau))\,d\tau$$

Determine also the closed loop state equation and the optimal trajectory. Is the closed loop system stable? What is the optimal cost?

Some formulas that might be useful:

$$\int_0^\infty \log |S(i\omega)|\,d\omega = \pi \sum_{i=1}^{M} \mathrm{Re}\,(p_i)$$

$$|W_T(p_1)| \le 1 \quad \Rightarrow \quad \omega_0 \ge \frac{p_1}{1 - 1/T_0}$$

$$|W_S(z)| \le 1 \quad \Rightarrow \quad \omega_0 \le (1 - 1/S_0)\,z$$

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2}x^T(T)S(T)x(T) + \frac{1}{2}\int_{t_0}^{t}\left(x^T Qx + u^T Ru\right)dt$$

$$S(T) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \le T, \quad \text{boundary condition } S(T)$$

$$K = R^{-1}B^T S$$

$$u = -Kx,$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$