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T-79.4202 Principles of Algorithmic Techniques (5 cr)
Exam Thu 19 Dec 2013, 1–4 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.4202 Principles of Algorithmic Techniques 19.12.2013"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Arrange the following functions according to their increasing order of growth: n, $\sqrt{n}$, $n^{1/n}$, $n\log_2\log_2 n$, $\sqrt{n}\log_2(n^2)$, $2/n$, 2^n, $2^{(\log_2 n)^2}$.

2. Recall that in Strassen's efficient divide-and-conquer algorithm for matrix multiplication, two $n \times n$ matrices are partitioned into $\frac{n}{2} \times \frac{n}{2}$ submatrices that are then multiplied together using 7 recursive calls, instead of 8 as would be suggested by a straightforward approach. This results in an algorithm with a running time of $O(n^{\log_2 7})$. Now suppose that you came up with an idea for multiplying 3×3 matrices together using $m < 27$ multiplications. What would be the running time of a divide-and-conquer algorithm for multiplying $n \times n$ matrices based on this idea? How small should the value of m be so that your algorithm would be asymptotically faster than Strassen's method? (It suffices to give a closed-form expression for the critical value of m; you do not need to actually compute the numerical result if you don't happen to have a calculator.)

3. Give an algorithm with running time $O(n+m)$ for the following task. The input is a set of n variables $x_1, x_2, \ldots, x_n$ and a set of m constraints, each of which is either an *equality* constraint of the form "$x_i = x_j$" or a *disequality* constraint of the form "$x_i \neq x_j$" for some $1 \leq i, j \leq n$. The task is to decide whether the variables can be assigned values so that all the constraints are satisfied. For example, it is not possible to satisfy the four constraints

$$x_1 = x_2,\ x_2 = x_3,\ x_3 = x_4,\ x_1 \neq x_4.$$

   Hint: Apply your knowledge of graph algorithms.

4. *Currency arbitrage* is the exploitation for profit of differences in foreign currency exchange rates. For example, if one Euro buys 9.10 Swedish Crowns, one Crown 0.15 US Dollars, and one Dollar 0.75 Euros, then by first exchanging Euros into Crowns, then Crowns into Dollars, and finally Dollars back into Euros it is possible to achieve a profit ratio of $9.10 \times 0.15 \times 0.75 = 1.02$. Design an efficient algorithm that determines if profitable arbitrage (i.e. with profit ratio > 1) can be achieved, given a matrix of exchange rates $R[i,j]$ of currency i into currency j, $1 \leq i, j \leq n$. (*Hint:* Floyd-Warshall.)

Grading: Each problem 12p, total 48p.