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T-79.5103 Computational Complexity Theory (5 cr)
First Midterm Exam, Mon 11 Feb 2014, 12–2 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 11.2.2014"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. (a) Design (i.e. give the transition diagram for) a Turing machine M that removes trailing 0's from a binary input string, i.e. computes the following function $f: \{0,1\}^* \longrightarrow \{0,1\}^*$:

$$f(x) = \begin{cases} y1 & \text{if } x = y10^k \quad \text{for some } y \in \{0,1\}^*,\ k \geq 0, \\ \varepsilon & \text{if } x = 0^k \quad \text{for some } k \geq 0. \end{cases}$$

   where ε denotes the empty string. (For instance, $f(10100) = 101$ and $f(000) = \varepsilon$.

   (b) Give the computation sequences of your machine, i.e. the lists of configurations the machine passes through until it halts, on inputs 010, 00, and ε.

2. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

   (a) The computation of a deterministic Turing machine halts on every input.

   (b) All languages accepted by deterministic Turing machines are recursive.

   (c) Nondeterministic Turing machines can accept also nonrecursive languages.

   (d) The complement of any language decided by a Turing machine is recursively enumerable.

   (e) The intersection of any two recursively enumerable languages is recursive.

   (f) The problem of determining if a Turing machine has at least 7 states is undecidable.

   (g) The problem of determining if a Turing machine accepts at least 7 strings is undecidable.

   (h) A problem A can be shown to be undecidable by devising a reduction mapping t from A to the Halting Problem.

3. (a) Define the formal language L_{101} representing the decision problem:

   Given a Turing machine M; does M accept *only* the string '101'?

   (b) Prove, without appealing to Rice's theorem, that the language L_{101} is not recursive.

Grading: Each problem 4p, total 12p.