Aalto University School of Science, Department of Information and Computer Science JR

T-79.3001 Logic in Computer Science: Foundations

Examination, February 20, 2014

Please note the following: your answers will be graded only if you have completed the three obligatory home assignments before the exam!

Assignment 1 $(1+2+7p)$

Two 3-bit binary numbers in the range 0 to 7 are represented by the sequences of propositional variables $x_2x_1x_0$ and $y_2y_1y_0$, where x_2 and y_2 are respectively the most significant bits of the numbers.

1. Construct a propositional formula which is true if and only if the value of the binary number $x_2x_1x_0$ is even.
2. Construct a propositional formula that is true if and only if binary numbers $x_2x_1x_0$ and $y_2y_1y_0$ are not equal.
3. Construct a propositional formula that is true if and only if $x_2x_1x_0 = y_2y_1y_0 + 001$ holds, that is, the first number is one less than the second number.

Assignment 2 $(2+3+5p)$

(a) Let ϕ_1 and ϕ_2 be sentences of the propositional logic. If ϕ_1 and ϕ_2 are satisfiable, is $(\phi_1 \vee \neg\phi_1) \wedge \phi_2$ satisfiable? Is it valid? Justify your answer.

(b) Let $A = \{a_1, \ldots, a_5\}$ be the atomic propositions. Give five formulas (in the propositional logic) that respectively have

1. 0 models,
2. 1 model,
3. 2 models,
4. 3 models, and
5. 2^5 models.

(c) Let ϕ be any sentence in the propositional logic that consists of atomic propositions as well as connectives $\vee$, $\wedge$ and $\neg$, with at most one occurrence of each atomic proposition in ϕ. Claim: ϕ is satisfiable. Is this claim necessarily true? Give a proof sketch or give a counter-example.

Assignment 3 $(10p)$ Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow B) \wedge (\neg A \rightarrow \neg B) \rightarrow (A \wedge \neg B \leftrightarrow B \wedge \neg A)$.

(b) $\models \forall x(\exists y R(x,y) \rightarrow P(x)) \rightarrow \forall y \forall x(P(x) \vee \neg R(x,y))$.

Tableau proofs must contain all intermediate steps !!!

Assignment 4 $(10p)$ Derive a Prenex normal form and a clausal form (i.e., a set of clauses S) for the sentence $\neg(\exists x(P(x) \vee \forall y Q(x,y)) \rightarrow \exists y(P(y) \vee Q(y,y)))$.

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 5 $(10p)$

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Double`:

```
v=0; z=x; while(!(z==0)) {z=z-1; v=v+2}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Double}\ [\texttt{v==2*x}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.