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OPEN-BOOK EXAM
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# S-72.3410 Coding Methods

1. A (7,4) Hamming code has the following set of codewords:

| | | | |
|---|---|---|---|
| 0000000 | 1101000 | 0110100 | 1011100 |
| 0011010 | 1110010 | 0101110 | 1000110 |
| 0001101 | 1100101 | 0111001 | 1010001 |
| 0010111 | 1111111 | 0100011 | 1001011 |

   (a) (2p.) Construct a parity-check matrix for this code.

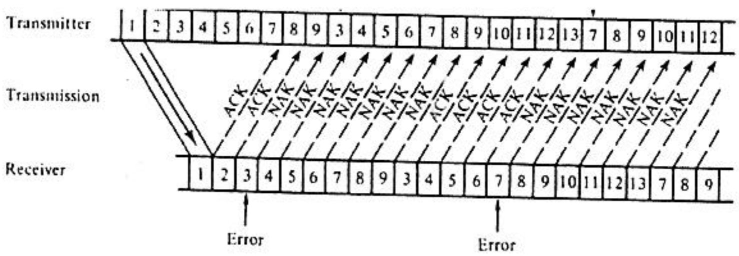
   (b) (2p.) Draw a Tanner graph corresponding to the parity-check matrix obtained in part (a).

   (c) (1p.) This code is also cyclic. What is the generator polynomial of this code?

   (d) (1p.) Show that this code is perfect.

2. (a) (3p.) Find the generator polynomial of a two-error-correcting binary cyclic code of length 63.

   (b) (3p.) Find the generator polynomial of a two-error-correcting 64-ary cyclic code of length 63. Express the coefficients of the generator polynomial as powers of a primitive element of GF(64). *Hint:* vector space representations of the elements of a certain Galois field may be useful here.




3. (a) (2p.) Identify the retransmission scheme depicted in the picture above. Describe its operation briefly.

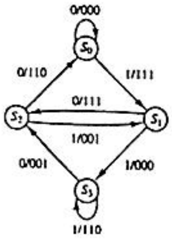
   (b) (4p.) Consider the ring $R_4 = \mathrm{GF}(2)[x]/(x^4 + 1)$.

   (i) How many elements are there in $R_4$? Justify your answer.

   (ii) Multiply the polynomials $a(x) = x^3 + x^2 + 1$ and $b(x) = x^2 + x$ in $R_4$.

   (iii) Find a non-trivial ideal in $R_4$. *Hint:* the smallest non-trivial ideal has only two elements; the theory of cyclic codes may be helpful here.

4. (6p.) Assume that we are using the rate-1/3 convolutional code whose encoder state diagram is pictured below over a binary symmetric memoryless channel.




Soft-decision Viterbi decoding is used, where 3-bit quantization is applied to the received noisy channel samples prior to decoding. We assume that the system can now be modeled as a binary-input, 8-ary output discrete symmetric channel, where the conditional probabilities $p(r|y)$ are shown in the following table:

| $p(r \mid y)$ | $r = 0_1$ | $r = 0_2$ | $r = 0_3$ | $r = 0_4$ | $r = 1_4$ | $r = 1_3$ | $r = 1_2$ | $r = 1_1$ |
|---|---|---|---|---|---|---|---|---|
| $y = 0$ | 0.434 | 0.197 | 0.167 | 0.111 | 0.058 | 0.023 | 0.008 | 0.002 |
| $y = 1$ | 0.002 | 0.008 | 0.023 | 0.058 | 0.111 | 0.167 | 0.197 | 0.434 |

Thus, for example, $0_1$ represents "strong 0", and so on. By using the transformation

$$M(r|y) = 5(\log_2 p(r|y) + 9),$$

we get (approximately) the following integer-valued bit metrics:

| $M(r \mid y)$ | $r = 0_1$ | $r = 0_2$ | $r = 0_3$ | $r = 0_4$ | $r = 1_4$ | $r = 1_3$ | $r = 1_2$ | $r = 1_1$ |
|---|---|---|---|---|---|---|---|---|
| $y = 0$ | 39 | 33 | 32 | 29 | 24 | 18 | 10 | 0 |
| $y = 1$ | 0 | 10 | 18 | 24 | 29 | 32 | 33 | 39 |

**Problem:** Use the soft-decision Viterbi algorithm to decode the received sequence

$$\mathbf{r} = (1_2 0_1 1_2, 0_3 0_1 1_3, 0_1 1_3 1_1, 1_2 0_2 0_2, 0_3 1_1 0_3, 0_3 0_2 0_1).$$