Quantum physics TFY-3.4323
19th February, 2014

Midterm exam

1. Explain briefly (you may use equations if you find it convenient, but it is not necessary)

   a) Interaction picture (2 points)
   b) Second quantization (2 points)
   c) Spontaneous emission and its connection to quantizing the electromagnetic field (2 points).

2. Derive the Fermi golden rule for the (absorption) transition rate from the initial state $|\psi(t=0)\rangle = |i\rangle$ to a continuous part of the spectrum.

   a) Start from the first-order time-dependent perturbation theory result

   $$\langle f|\psi(t)\rangle \approx \frac{1}{i\hbar}\int_0^t dt' \, \langle f|H'(t')|i\rangle e^{i\omega_{fi}t'},$$

   where $|i\rangle$ and $|f\rangle$ are the initial and final states of the system, $\omega_{fi} = (E_f - E_i)/\hbar$, and $H'(t) = Ae^{-i\omega t} + A^\dagger e^{i\omega t}$ is a harmonic perturbation. Derive the result for resonant absorption probability

   $$P_{if}(t) = \frac{1}{\hbar^2}|\langle f|A|i\rangle|^2 \left[\frac{\sin\left[\frac{1}{2}(\omega - \omega_{fi})t\right]}{\frac{1}{2}(\omega - \omega_{fi})}\right]^2, \quad \text{for} \quad \omega \approx \omega_{fi},$$

   You can assume that the perturbation is long lasting, such that $|\omega t| \gg 1$.

   b) Using the above transition probability, write the total transition rate out from the initial state $|i\rangle$ when there are several possible final states $|f_n\rangle$ with energies $E_{f,n} = \hbar\omega_n$. You can assume that the coupling $\langle f|A|i\rangle$ is equal for all final states $|f\rangle = |f_n\rangle$. Now write the total transition rate when the final states form a continuum with constant density of states $g(E) = g$.

   c) Solve the total transition rate and derive the Fermi golden rule. Since off-resonant (non-energy-conserving) processes are strongly suppressed, you are able to do some approximations. The following relation may also prove useful

   $$\int_{-\infty}^{\infty}\left[\frac{\sin x}{x}\right]^2 dx = \pi.$$

TURN THE PAPER!