The exam is three hours long and consists of 4 exercises. The exam is graded on a scale 0-25 points, and the points assigned to each question are indicated in parenthesis within the text. If necessary, use separate sheets as scratch papers, but write your final answers in the exam paper clearly, and explain your solutions.

Problem 1

Solve the following problem using the Two-Phase Simplex method. Use Bland's rule as a pivoting rule. Report the tableau at each iteration for both phases and clearly indicate entering and leaving variables. (6pt) Is the optimal primal solution degenerate? (1pt)

$$\begin{aligned} \min \quad & -4x_1 - 5x_2 + 3x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 = 10 \\ & x_1 - x_2 \geq 6 \\ & x_1 + 3x_2 + x_3 \leq 14 \\ & x_1, x_2, x_3 \geq 0 \ . \end{aligned}$$

Problem 2

Consider a minimization linear programming problem P in standard form defined by an $m \times n$ constraint matrix $\mathbf{A}$, a cost vector $\mathbf{c}$, and a right hand side vector $\mathbf{b}$. Assume that the rows of $\mathbf{A}$ are linearly independent. Are the following statements true or false? Justify your answers. (5pt)

(a) Let $\mathbf{x}$ be a basic feasible solution of P, and let $\mathbf{p}$ be the associated basic dual solution. Then, $\mathbf{p}$ and $\mathbf{x}$ satisfy the complementary slackness conditions.

(b) If the polyhedron $\{\mathbf{x} \in \mathbb{R}^n | \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ is unbounded, then the optimal cost of P is $-\infty$.

(c) Let $\mathbf{x}^*$ be a basic feasible solution. Suppose that for every basis corresponding to $\mathbf{x}^*$, the associated basic dual solution is infeasible. Then, the optimal cost must be strictly less than $\mathbf{c}^T\mathbf{x}^*$.

(d) Let p_i be the dual variable associated with the i-th equality constraint of P. Eliminating the i-th equality constraint of P is equivalent to introducing the additional constraint $p_i = 0$ in the dual of P.

(e) If P is solved by the Simplex method, the value of the objective function strictly decreases at each iteration of the Simplex algorithm.