Problem 3

Consider the following LP and its optimal tableau below:

$$\begin{aligned} \min \quad & -2x_1 - x_2 + x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \leq 8 \\ & -x_1 + x_2 - 2x_3 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \,. \end{aligned}$$

		x_1	x_2	x_3	x_4	x_5
	16	0	3	3	2	0
$x_1 =$	8	1	2	1	1	0
$x_5 =$	12	0	3	-1	1	1

(a) Write the dual and obtain the optimal dual variables (e.g., you can use complementary slackness, or note that in this case the duals can be derived directly from the tableau). Explain how and why. (2pt)

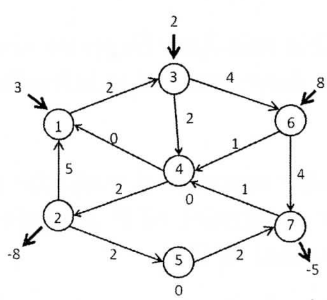
(b) If you were to choose between increasing of 1 unit the right hand side of the first and second constraint, which one would you choose, and why? What is the effect of the increase on the optimal cost? (1pt)

(c) Suppose the constraint $x_1 + 2x_2 + x_3 \leq 8$ is replaced by $x_1 + \frac{1}{6}x_2 + x_3 \leq 8$. Does the current basis remain optimal? Justify. (1pt)

(d) Suppose the following constraint is added to the problem: $x_2 + 2x_3 \geq 3$. Find a new optimal solution starting from the tableau above. (3pt)

Problem 4

Consider the uncapacitated min cost flow problem defined by the graph below. The number on each arc indicates its cost, and node supplies are indicated by the bold arrows.



(a) Solve the problem using the Network Simplex. Start from the tree solution defined by arcs $T = \{(1,3), (3,6), (6,7), (7,4), (4,2), (2,5)\}$. Explain all the steps. At each iteration, draw the graph with the updated flows, indicate the set T, and report the dual variables. (5pt)

(b) Suppose the supply at node 1 is changed to 4, and the supply at node 7 is changed to -6. Is the optimal basis at the end of point (a) still optimal after the change? Justify. (1pt)