Aalto University
School of Science and Engineering
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Spring 2010

Examination
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1. Consider a queueing system with 6 parallel servers and 4 waiting places. The average service time is 3 min. Customers are served in their arrival order. Assume an excessive arrival stream, that is, every time a customer leaves the system, a new customer arrives immediately. Therefore the system is always full. What is the average time a customer spends in the system?

2. Consider the M/M/1/2/2 model where the mean idle time of a customer is $1/\nu$ time units and the mean service time is $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t.

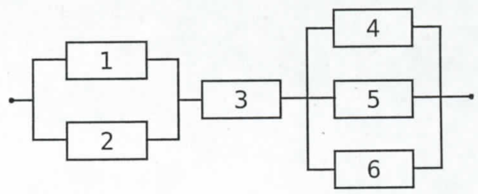
   (a) Draw the state transition diagram of the Markov process $X(t)$.

   (b) Derive the equilibrium distribution of $X(t)$.

   (c) Assumed that $\nu = \mu$, determine the time blocking probability and the call blocking probability.

3. Consider a queueing system with two parallel servers. The service times are independent and identically distributed following the Exp(1) distribution. The system is empty at time 0. Two new customers arrive at times 1 and 2, respectively. No other customers enter the system. In addition, it is known that both customers are still in the system at time 3. Let Z_1 denote the time at which the customer with the shorter service time leaves the system. Correspondingly, let Z_2 denote the time at which the customer with the longer service time departs. Thus, $Z_2 > Z_1 > 3$. Determine the mean values $E[Z_1]$ and $E[Z_2]$.

4. (a) Determine the structure function $\phi(\mathbf{x})$ of the structure of independent components in the reliability block diagram below.

   (b) If the components in above diagram are repairable, what is the availability of the above system? The availability of each of the components 1 and 2 is 2/3, the availability of component 3 is 1 and the availability of each of the components 4, 5 and 6 is 1/2?

   (c) Tell briefly in words what is **availability**? (That is, give some definiton of availability)

5. a) Assume the you can use a pseudo random number generator to easily generate samples of the random variable U that is uniformly distributed between $(0, 1)$, i.e., $U \sim \mathrm{U}(0, 1)$. Apply the inverse transform method to generate samples of the random variable X obeying Exp(λ) distribution (exponential distribution with mean $1/\lambda$).

   b) Again assume that you have a pseudo-random number generator to generate samples of $U \sim \mathrm{U}(0, 1)$. Give a pseudo code description for simulating a Poisson arrival process with intensity λ to count the number of arrivals between the time $[0, T]$.