Aalto University
School of Science and Engineering
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Spring 2010

Examination
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1. Buses arrive at a bus stop according to a Poisson process with an average interarrival time of 10 minutes. You arrive at the bus stop just after the departure of the previous bus.

   (a) Let T denote the time until the arrival of the next bus. What is the distribution of the random variable T?

   (b) Let X denote the number of buses that arrive during the next 5 minutes after your arrival. What is the distribution of the random variable X?

2. Consider the M/M/2/3 model with mean customer interarrival time of $1/\lambda$ time units and mean service time of $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t.

   (a) Draw the state transition diagram of Markov process $X(t)$.

   (b) Derive the equilibrium distribution of $X(t)$.

   (c) Assume that $\lambda = \mu$. What is the probability that an arriving customer is lost?

3. Consider a lossy queueing system with 2 parallel servers and 2 waiting places. The average interarrival time between two customers is 6 minutes, and the loss ratio is 10%. In addition, the average waiting time (before service) is 2 minutes, and the average service time is 8 minutes.

   (a) What is the average number of waiting customers?

   (b) What is the average number of customers in service?

4. A trunk network link can break down independently at either end point. The failure times at end point 1 are exponentially distributed with parameter λ_1 and repair times are also exponential with parameter μ_1. Similarly parameters λ_2 and μ_2 for end point 2. Both end points can be down at the same time.

   (a) Draw the reliability block diagram of the system and determine the structure function of the system.

   (b) Make a Markov model of the system and calculate the average availability.

5. a) Assume the you can use a pseudo random number generator to easily generate samples of the random variable U that is uniformly distributed between $(0, 1)$, i.e., $U \sim \mathrm{U}(0, 1)$. Apply the inverse transform method to generate samples of the random variable X obeying $\mathrm{Exp}(\lambda)$ distribution (exponential distribution with mean $1/\lambda$).

   b) Again assume that you have a pseudo-random number generator to generate samples of $U \sim \mathrm{U}(0, 1)$. Give a pseudo code description for simulating a Poisson arrival process with intensity λ to count the number of arrivals between the time $[0, T]$.