

Please answer all questions.

- ① What is a pseudo-random number generator and what is its role in discrete event simulation? What are the most important properties of a good random generator? Describe the Linear Congruential Generator (LCG) algorithm. (6p)
- ② The random variable X obeys the exponential distribution with mean 1. Thus, the cumulative distribution function is given by $F(x) = 1 - e^{-x}$. Assume that Y has the same distribution as X but conditioned on that $X \leq 2$, i.e., $P\{Y \leq y\} = P\{X \leq y | X \leq 2\}$. Show how one can generate values of Y from this distribution using a) the inverse transform (inverse distribution function) method and b) the acceptance-rejection method. Hint: $P\{A | B\} = P\{A \cap B\} / P\{B\}$. (6p)
- ③ Describe the discrete event simulation of a simple finite M/M/1/K FIFO queue, when the objective is to simulate the evolution of the queue length $X(t)$ starting from an empty system at time $t = 0$, $X(0) = 0$, until time $t = T$. In an M/M/1/K FIFO queue, the arrivals occur according to a Poisson process with intensity λ , the service times are exponentially distributed with mean $1/\mu$, there is one server and a *finite* number of waiting places, i.e., $K - 1$, and the service discipline is FIFO. Give the state variables, events and how the values of the required random variables are generated. Describe the logic of the simulation, i.e., the handling of the events by using suitable pseudo-code.
4. a) Describe different methods used in steady state simulation that can be used to obtain (nearly) independent samples for estimation of the confidence interval? Discuss also the handling of the initial transient. (5p)
b) Assume that K independent samples, X_1, \dots, X_K , are available. As the final estimate for the mean, \bar{X} , one uses

$$\bar{X} = \frac{1}{K} \sum_{k=1}^K X_k.$$

Give the 95% confidence intervals for \bar{X} . If the samples X_k are positively correlated, how does it affect the width of the confidence interval? Justify! (4p)

- ⑤ Consider a system that can be operated using two different policies, A and B. The difference in the performance of the policies can be estimated by simulating independently the system under policy A and B, respectively. Give a method with which the difference in performance can be performed more efficiently. Justify the obtained efficiency gain. (3p)