ELEC-A3110 Mechanics First mid term exam Sep 25, 2013

Allowed material: writing implements and a graphing calculator. You are not allowed to use any other material. There are some formulas and constants tabulated in last page of the exam. Justify the formulas you use in your answers and introduce the meaning of the symbols within these formulas. Solve each problem on separate page.

It is important that you at least try each problem. Good luck!

1. Define the following terms using at most about 30 words. Answering with only formulas will not yield full points. a) angular acceleration b) Coriolis acceleration c) inertial frame d) uniform circular motion e) position vector f) Galilean transformation

2. The speed of a rifle bullet is 1.0 km/s as it exits the rifle barrel. Let the barrel length be 100 cm.

   (a) What is the acceleration of the bullet as it travels through the barrel?

   (b) How many milliseconds does the bullet spend traveling through the barrel?

   In both parts use averaged quantities. Specify the essential assumptions you made when solving the problem and include them in your solution.

3. The trajectory of a particle in xy-plane is described by equations

   $$x(t) = R(\omega t - \sin \omega t) \text{ ja } y(t) = R(1 - \cos \omega t)$$

   where R and ω are constants. The trajectory is called cycloid ja it describes the position of a point in wheel rim as it travels at constant speed horizontally.

   Determine a) the components of the velocity and acceleration of the particle as a function of time, b) the points in time and space, where the particle is stationary, and the magnitude of the particle's total acceleration.

Write CLEARLY in each paper your name, student number, degree programme, the code of the study module, and the date of the exam. Solve each problem on separate page.

Kaavoja – Formulas – Formler

$$F_{\text{net}} = ma_{rad} = m\frac{v^2}{R}$$

$$v' = v + v_{\text{AB}}$$

$$\boldsymbol{p} = m\boldsymbol{v}$$

$$\omega = 2\pi f$$

$$a_T = r\alpha$$

$$a' = a$$

$$a = \sqrt{a_T^2 + a_N^2}$$

$$v(t) = v_0 + a_{\text{ave}}t$$

$$v_{\text{ave}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$2a_0(x - x_0) = v^2 - v_0^2$$

$$a_T = \frac{dv}{dt}$$

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\boldsymbol{w} = m\boldsymbol{g}$$

$$a_N = \frac{v^2}{R}$$

$$f_s \leq \mu_s N$$

$$\boldsymbol{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}}$$

$$f_k = \mu_k N$$

$$a = \frac{dv}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

$$\boldsymbol{f} = -k\boldsymbol{v}$$

$$v = r\omega$$

$$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}$$

$$a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$s = r\theta$$

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$$

$$f_r = \mu_r N$$

$$\alpha_{\text{ave}} = \frac{\Delta\omega}{\Delta t}$$

$$\boldsymbol{a} = \frac{d^2\boldsymbol{r}}{dt^2}$$

$$r' = r + v_{\text{AB}}t$$

$$\boldsymbol{f} = -Dv^2\boldsymbol{e}_T$$

$$v = \frac{dx}{dt}$$

Fundamental Constants

Constant	Symbol	Value
Velocity of light	c	2.9979×10^{8} m s^{-1}
Elementary charge	e	1.6021×10^{-19} C
Electron rest mass	m_e	9.1091×10^{-31} kg
Proton rest mass	m_p	1.6725×10^{-27} kg
Neutron rest mass	m_n	1.6748×10^{-27} kg
Planck constant	h	6.6256×10^{-34} J s
	$\hbar = h/2\pi$	1.0545×10^{-34} J s
Charge-to-mass ratio for electron	e/m_e	1.7588×10^{11} kg^{-1} C
Quantum charge ratio	h/e	4.1356×10^{-15} J s C^{-1}
Bohr radius	a_0	5.2917×10^{-11} m
Compton wavelength:		
of electron	$\lambda_{C,e}$	2.4262×10^{-12} m
of proton	$\lambda_{C,p}$	1.3214×10^{-15} m
Rydberg constant	R	1.0974×10^{7} m^{-1}
Bohr magneton	μ_B	9.2732×10^{-24} J T^{-1}
Avogadro constant	N_A	6.0225×10^{23} mol^{-1}
Boltzmann constant	k	1.3805×10^{-23} J K^{-1}
Gas constant	R	8.3143 J K^{-1} mol^{-1}
Ideal gas normal volume (STP)	V_0	2.2414×10^{-2} m^3 mol^{-1}
Faraday constant	F	9.6487×10^{4} C mol^{-1}
Coulomb constant	K_e	8.9874×10^{9} N m^2 C^{-2}
Vacuum permittivity	ϵ_0	8.8544×10^{-12} N^{-1} m^{-2} C^2
Magnetic constant	K_m	1.0000×10^{-7} m kg C^{-2}
Vacuum permeability	μ_0	1.2566×10^{-6} m kg C^{-2}
Gravitational constant	γ	6.670×10^{-11} N m^2 kg^{-2}
Acceleration of gravity at sea level and at equator	g	9.7805 m s^{-2}

Numerical constants: $\pi = 3.1416$; $e = 2.7183$; $\sqrt{2} = 1.4142$; $\sqrt{3} = 1.7320$