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MS-C1340 Lineaarialgebra ja differentiaaliyhtälöt

1st midterm exam 18.11.2013

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark the course code, title and whether the exam is a mid-term exam or a final exam. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

1. Give a definition for the following notions.
   (a) Linear map
   (b) Norm
   (c) Orthonormal basis

2. Show that

$$\left\{ \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}x, \frac{3}{2}\sqrt{\frac{5}{2}}\left(x^2 - \frac{1}{3}\right) \right\}$$

is an orthonormal basis of the space $\mathbb{P}_2$ (=the set of real polynomials of degree 2 at most), where the inner product is given by

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx.$$

3. A linear map $A : \mathbb{R}^3 \to \mathbb{R}^3$ satisfies

$$A(1,0,0) = (2,1,1)$$
$$A(0,1,0) = (4,2,1)$$
$$A(0,0,1) = (8,4,1).$$

Find bases for the image and the null space of A. Are the columns of the matrix of A linearly independent?

4. Let V be a vector space with real coefficients and an inner product $\langle \cdot, \cdot \rangle$. The *orthogonal complement* of a set $S \subset V$ is defined by

$$S^\perp = \{\boldsymbol{w} \in V \mid \langle \boldsymbol{v}, \boldsymbol{w} \rangle = 0 \text{ for all } \boldsymbol{v} \in S\}.$$

   (a) Show that $S^\perp$ is a vector subspace of V.
   (b) Assume that $\dim(V) < \infty$. Show that $(S^\perp)^\perp = \mathrm{sp}(S)$, where $\mathrm{sp}(S)$ is the set of all linear combinations of the elements of S.