Tfy-0.3223 Statistical Physics

Exam 11.12.2013

1. *Two-state system.* Consider N impurity atoms trapped in a solid lattice. Each impurity can have two energies, 0 and ϵ. Calculate entropy, equilibrium temperature and heat capacity $C = dE/dT$ of the impurities.

2. Consider the statistics of a non-interacting gas of particles on energy states E_ℓ. On any given state, there can be up to p particles, i.e. the allowed occupation numbers are $n_\ell = 0, 1, \ldots, p$. The total energy of the system is then $E = \sum_\ell n_\ell E_\ell$. (These hypothetical particles are called anyons.)

   (a) Calculate the grand canonical partition function.

   (b) Calculate the average occupation number $\langle n_\ell \rangle$.

   (c) Consider the limits $p = 1$ and $p \to \infty$. What is $\langle n_\ell \rangle$ in these limits, and what are the corresponding statistics?

   Hint: use the expression for finite geometric series:

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x} \tag{1}$$

3. Use the canonical formalism to derive the heat capacity of a set of $3N$ quantum harmonic oscillators. Show that it reduces to the classical result $C_N = 3Nk_B$ in the limit $T \to \infty$.

4. Consider a 1D random walker that moves one step to the right with probability p_1, two steps to the right with probability p_2 and one step to the left with probability q on an infinite lattice with lattice spacing of L (see Fig. 1).

   (a) What is the condition that q, p_1 and p_2 must satisfy? (1 p.)

   (b) What is the condition that q, p_1 and p_2 must satisfy for there to be no particle drift (i.e. $\langle \Delta x \rangle$=0) after N steps ($N \to \infty$)? (2 p.)

   (c) Calculate the tracter diffusion coefficient when there is no drift. (3 p.)

   Recall: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$

TURN

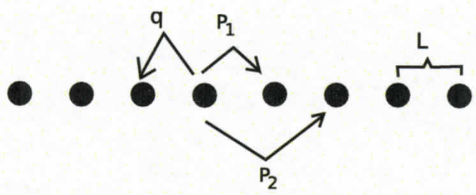

Figure 1: Random walk of question 4.