Aalto University
School of Electrical Engineering
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Autumn 2013

Examination
20.1.2014
Lassila

Please answer to all five (5) questions

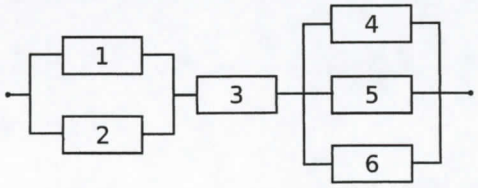
1. Buses arrive at a bus stop according to a Poisson process with an average interarrival time of 20 minutes. You arrive at the bus stop at a random time. Thus, you don't know when the previous bus left, nor when the next bus will arrive.
   (a) Let T denote the time until the arrival of the next bus. Specify the distribution and the mean value of the random variable T?
   (b) What is the probability that at least one bus arrives during the 10-minute interval following your arrival?

2. Consider the M/M/1/2 model with mean customer interarrival time of $1/\lambda$ time units and mean service time of $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t.
   (a) Draw the state transition diagram of the Markov process $X(t)$.
   (b) Derive the equilibrium distribution of $X(t)$. Are there any stability conditions?
   (c) Assume that $\lambda = 2\mu$. What is the probability that the service of an arriving customer is started immediately upon the arrival (without any wait)?

3. Consider a statistical multiplexer with three incoming links. The average packet arrival rates from these three links are as follows: 3, 4, and 2 packets/millisecond. The average packet length is 10 000 bits. Packets arrive to the multiplexer from each incoming link according to independent Poisson processes. By using the teletraffic model of type $M/M/1$, answer the following questions.
   (a) What is the minimum capacity required for the outgoing link in order that the system be stable?
   (b) Assume then that the capacity of the outgoing link is 100 Mbps. What is the traffic load? What is the average number of packets in the multiplexer? What is the mean delay of a packet (from its arrival to the end of its transmission)?

4. Consider a Markov process with state space $S = \{0, 1, 2\}$ and the following state transition diagram
$$Q = \begin{pmatrix} - & 1 & 0 \\ 0 & - & 1 \\ 1 & \mu & - \end{pmatrix}.$$
   Draw the state transition diagram of the process. Is this process irreducible? Write and solve the global balance equations (GBE).

5. (a) Determine the structure function $\phi(\mathbf{x})$ of the structure of independent components in the reliability block diagram below.



   (b) If the components in above diagram are repairable, what is the availability of the above system? The availability of each of the components 1 and 2 is 2/3, the availability of component 3 is 1 and the availability of each of the components 4, 5 and 6 is 1/2?