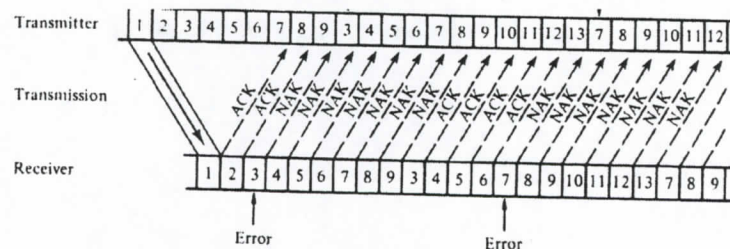


S-72.3410 Coding Methods

1. A (7,4) Hamming code has the following set of codewords:

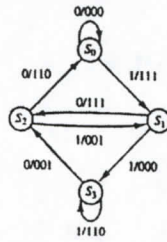
0000000 1101000 0110100 1011100
 0011010 1110010 0101110 1000110
 0001101 1100101 0111001 1010001
 0010111 1111111 0100011 1001011

- (a) (2p.) Construct a parity-check matrix for this code.
 - (b) (2p.) Draw a Tanner graph corresponding to the parity-check matrix obtained in part (a).
 - (c) (1p.) This code is also cyclic. What is the generator polynomial of this code?
 - (d) (1p.) Show that this code is perfect.
2. (a) (3p.) Find the generator polynomial of a two-error-correcting binary cyclic code of length 63.
- (b) (3p.) Find the generator polynomial of a two-error-correcting 64-ary cyclic code of length 63. Express the coefficients of the generator polynomial as powers of a primitive element of $GF(64)$. *Hint:* vector space representations of the elements of a certain Galois field may be useful here.



3. (a) (2p.) Identify the retransmission scheme depicted in the picture above. Describe its operation briefly.
- (b) (4p.) Consider the ring $R_4 = GF(2)[x]/(x^4 + 1)$.
- (i) How many elements are there in R_4 ? Justify your answer.
 - (ii) Multiply the polynomials $a(x) = x^3 + x^2 + 1$ and $b(x) = x^2 + x$ in R_4 .
 - (iii) Find a non-trivial ideal in R_4 . *Hint:* the smallest non-trivial ideal has only two elements; the theory of cyclic codes may be helpful here.

4. (6p.) Assume that we are using the rate-1/3 convolutional code whose encoder state diagram is pictured below over a binary symmetric memoryless channel.



Soft-decision Viterbi decoding is used, where 3-bit quantization is applied to the received noisy channel samples prior to decoding. We assume that the system can now be modeled as a binary-input, 8-ary output discrete symmetric channel, where the conditional probabilities $p(r|y)$ are shown in the following table:

$p(r y)$	$r = 0_1$	$r = 0_2$	$r = 0_3$	$r = 0_4$	$r = 1_4$	$r = 1_3$	$r = 1_2$	$r = 1_1$
$y = 0$	0.434	0.197	0.167	0.111	0.058	0.023	0.008	0.002
$y = 1$	0.002	0.008	0.023	0.058	0.111	0.167	0.197	0.434

Thus, for example, 0_1 represents "strong 0", and so on. By using the transformation

$$M(r|y) = 5(\log_2 p(r|y) + 9),$$

we get (approximately) the following integer-valued bit metrics:

$M(r y)$	$r = 0_1$	$r = 0_2$	$r = 0_3$	$r = 0_4$	$r = 1_4$	$r = 1_3$	$r = 1_2$	$r = 1_1$
$y = 0$	39	33	32	29	24	18	10	0
$y = 1$	0	10	18	24	29	32	33	39

Problem: Use the soft-decision Viterbi algorithm to decode the received sequence

$$\mathbf{r} = (1_2 0_1 1_2, 0_3 0_1 1_3, 0_1 1_3 1_1, 1_2 0_2 0_2, 0_3 1_1 0_3, 0_3 0_2 0_1).$$