Aalto University School of Electrical Engineering
Department of Communications and Networking

EXAM
17.03.2014

S-72.2410 Information Theory

1. (6p.) Concepts and terminology. Define the following terms in a concise and precise way. Use words, not mathematical expressions or pictures.

   (a) (1p.) multiple access channel

   (b) (1p.) entropy

   (c) (1p.) universal coding

   (d) (1p.) SNR

   (e) (2p.) lossy vs. lossless compression

2. (6p.) Entropy. Consider two random binary variables, X and Y, describing the gender of a student in our school and the result of an exam. That is, the alphabets of the variables are $\mathcal{X} = \{\text{male}, \text{female}\}$ and $\mathcal{Y} = \{0, 1, 2, 3, 4, 5\}$. Moreover, $p(X = \text{female}) = 0.1$ and the probabilities $p(Y = i|X = \text{female})$ and $p(Y = i|X = \text{male})$ for $i = 0, 1, \ldots, 5$ are $(0, 0.1, 0.2, 0.4, 0.2, 0.1)$ and $(0.1, 0.15, 0.25, 0.25, 0.15, 0.1)$, respectively.

   (a) (2p.) Determine $p(X = \text{male}|Y = 4)$.

   (b) (2p.) Determine $H(X)$ and $H(Y)$.

   (c) (2p.) Determine $H(X|Y)$ and $I(X;Y)$.

3. (6p.) Source coding.

   (a) Design a binary prefix code for a source with alphabet $\{a, b, c, d, e, f\}$ and the following codeword lengths:

symbol	a	b	c	d	e	f
length	4	3	1	3	4	3

   (b) Consider the following Huffman code obtained for a source with the alphabet $\{a, o, e, i, u, y\}$ and probabilities as given (so, for example $p(a) = 0.3$):

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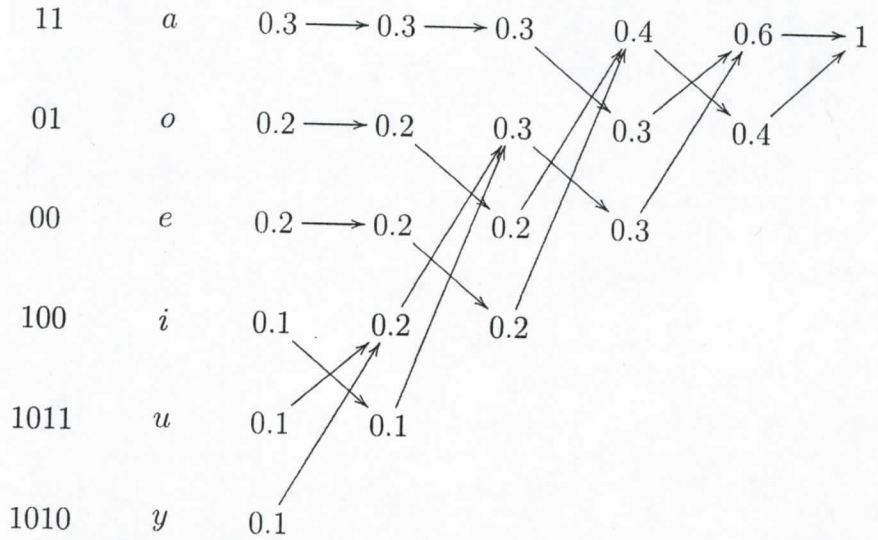

As you can see, two of the codewords have length 4. For this source, is it possible to design a Huffman code all of whose codewords have length smaller than 4 ? If no, motivate why. If yes, design such a code.

4. (6p.) Channel capacity. Consider the transmission system in Figure 1. The binary symmetric channel (BSC) has crossover probability p and input and output alphabet $\{0, 1\}$. The observer indicates $Z = 0$ whenever $X = Y$ and $Z = 1$ otherwise.

   (a) Determine the entropy $H(Z)$.

   (b) Show that X and Z are independent.

   (c) What is the maximal achievable rate of the system if the receiver is provided with both Y and Z ? Motivate.

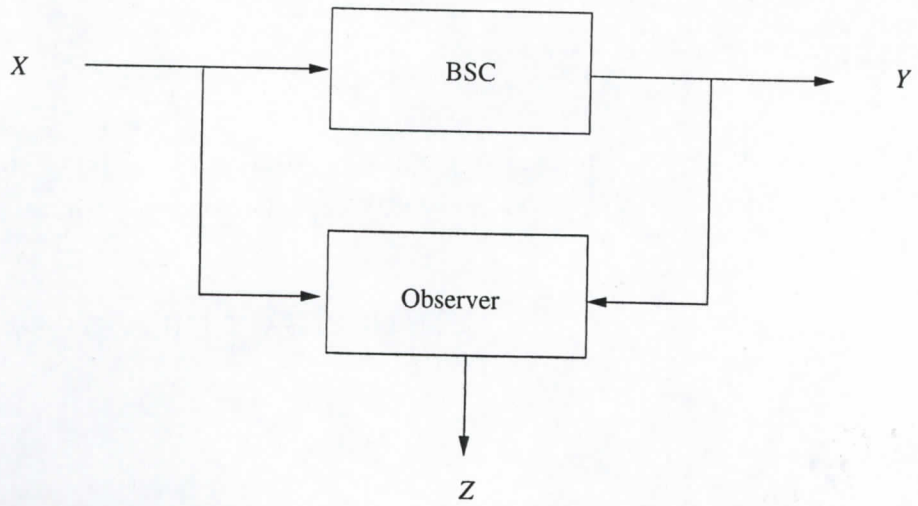

Figure 1: Channel