

BECS-114.1311 Introduction to Bayesian statistics

Examination
18.2.2014

1. Bayesian and frequentist statistical inference. Describe the basic principles of Bayesian and frequentist statistical modeling, and the differences and similarities between the methods (*Explanations have to be accurate enough to be interpretable without any guessing*). (6p)
2. Explain in a brief but accurate manner the following terms related to statistical inference and modeling
 - (a) Conditional distribution (1p)
 - (b) Conjugate prior (1p)
 - (c) Bias of an estimator (1p)
 - (d) Epistemic uncertainty (1p)
 - (e) Standard deviation (1p)
 - (f) Likelihood function (1p)
3. Assume that the number of claims received by an insurance company follows a $Poisson(\mu)$ distribution. The weekly number of claims observed over a ten week period are:

x_1	5	x_6	7
x_2	9	x_7	8
x_3	4	x_8	10
x_4	9	x_9	4
x_5	11	x_{10}	4

- (a) Formulate a Bayesian model. The prior distribution of the parameter μ has an expected value 6 and standard deviation 2. Justify your choice of the prior distribution. (2p)
 - (b) **Derive** the posterior distribution of μ . (2p)
 - (c) Calculate the 95% credible interval for the parameter μ by using a normal distribution approximation. (2p)
4. You are given observations $\{(x_i, y_i)\}_{i=1}^5$:

x	y
-3	-7.1
-2	-3.2
0	-1.8
1	2.9
4	9.1

You wish to explain the output variable y with an input variable x . Use a linear regression model $y_i = \beta x_i + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma^2)$, $i = 1, \dots, 5$. It is also known that $\sigma^2 = 1$.

- (a) Formulate a Bayesian linear regression model when it is assumed that the prior distribution of the unknown slope is of the form $\beta \sim N(0, \tau^2)$, with $\tau^2 = 2$ assumed to be known. Solve the posterior distribution of the slope β . (2p)
- (b) Solve the posterior predictive distribution for the future observation (\tilde{x}, \tilde{y}) , when $\tilde{x} = 5$. (2p)
- (c) Define and solve some Bayesian point estimate of your choice for the unknown slope. Describe how and why the prediction made with the posterior predictive distribution differs from the prediction made by using the point estimate. (2p)

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Useful formulas.

- Binomial distribution

$$f(y|\pi) = \binom{n}{y} \pi^y (1 - \pi)^{n-y}$$

- $E[y] = n\pi$
- $Var[y] = n\pi(1 - \pi)$

- Poisson distribution

$$f(y|\mu) = \frac{\mu^y e^{-\mu}}{y!}$$

- $E[y] = \mu$
- $Var[y] = \mu$

- Hypergeometric distribution

$$f(y|N, R, n) = \frac{\binom{R}{y} \binom{N-R}{n-y}}{\binom{N}{n}}$$

- $E[y] = n \frac{R}{N}$
- $Var[y] = n \frac{R}{N} (1 - \frac{R}{N}) (\frac{N-n}{N-1})$

- Gamma distribution, $0 \leq x < \infty$

$$f(x|r, v) = \frac{v^r}{\Gamma(r)} x^{r-1} e^{-vx},$$

- $E[x] = \frac{r}{v}$
- $Var[x] = \frac{r}{v^2}$

- Beta distribution

$$f(x|a, b) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x \notin [0, 1] \end{cases}$$

- $E[x] = \frac{a}{a+b}$
- $Var[x] = \frac{ab}{(a+b)^2(a+b+1)}$

- Normal distribution, $-\infty < x < \infty$

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

- $E[x] = \mu$
- $Var[x] = \sigma^2$

