ELEC-A7200 Signals and Systems

Exam 03.02.2014

Answer question 1, of the questions 2 – 6 the four best performed are taken into account.

1.

Give short answers to the following tasks, use figures when needed.

a) What are the conditions under which the set of functions $\{\phi_k(t)\}$ are orthonormal?

b) We want to determine the spectrum of a lowpass signal with 100 MHz bandwidth using DFT. Determine the required number of samples and the sampling interval such that the frequency resolution of the spectrum is 100 kHz.

c) Define settling time for lowpass filter.

d) What are the conditions for a stochastic signal to be wide sense stationary and ergodic?

e) Provide the equation for FM-signal when the modulating signal is $x(t)$, frequency deviation is Δf and carrier frequency is f_c.

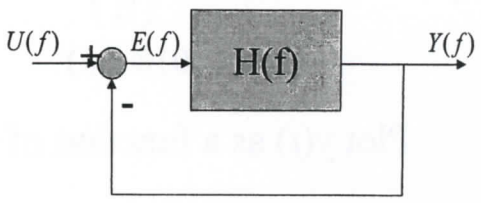
f) A signal consists of 10 kHz and 24 kHz sinusoidal components. The signal is sampled with 24 kHz sampling frequency. Determine the frequency components that appear in the sampled signal (0…24 kHz range).

g) Provide definition for total harmonic distortion.

h) Noise power density is -174 dBm/Hz. What is the noise power on 100 MHz band?

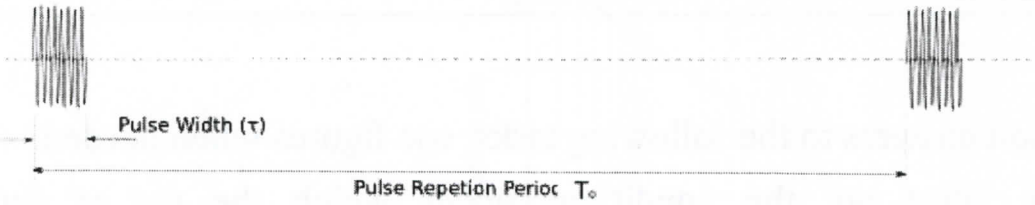
i) Consider a 12 bit A/D converter. Determine the quantization noise power when the input signal voltage can vary over the range [0V, 10V].

j) Determine the transfer function $Y(f)/U(f)$ of the feedback system shown in the figure.



2.

Consider a radar system that generates pulses of length τ=1 μs at pulse repetition rate f_{PRF}=1/T_0= 1 kHz. The peak transmit power is P 3 kW and the carrier frequency is 5.615 GHz.



Tutkan lähettämän signaalin lauseke on

$$s(t) = \sqrt{P} \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t - kT_0}{\tau}\right) \cos(2\pi f_c t)$$

$$\text{rect}(t) = \begin{cases} 1 & |t| \le \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$$

a) Determine the average power of the radar.

b) Provide the line spectra of the radar signal

3.

Determine the convolution $y(t)$ of two rectangular pulses $x_1(t)$ and $x_2(t)$ when $T_1 > T_2$.

$$x_1(t) = \text{rect}\left(\frac{t}{T_1}\right)$$

$$x_2(t) = \text{rect}\left(\frac{t}{T_2}\right)$$

$$y(t) = x_1(t) \otimes x_2(t)$$

Plot y(t) as a function of time.

4. Gaussian filter is used e.g. in GMSK-modulator. Its frequency response is given by

$$H(f) = \exp\left(-\frac{\ln 2}{2}\left(\frac{f}{B}\right)^2\right)$$

Determine the impulse response of Gaussian filter $h(t)$.

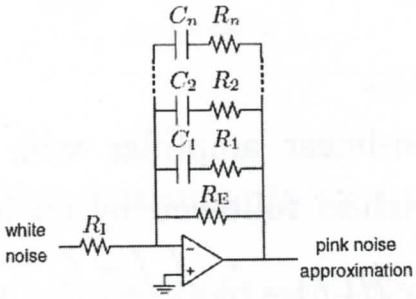
5. A band-pass system consists of a non-linear amplifier with the characteristic curve $y = 10x - 0{,}1x^3$, which is followed by an ideal band-pass filter with the transfer function $H(f) = rect\left(\frac{f - f_o}{B}\right)$ with $f_o = 1$ MHz and $B = 400$ kHz. The input signal is the sum of two cosine waves $x(t) = \cos(2\pi f_{x1} t) + \cos(2\pi f_{x2} t)$, $f_{x1} = 1{,}05$ MHz, $f_{x2} = 0{,}95$ MHz

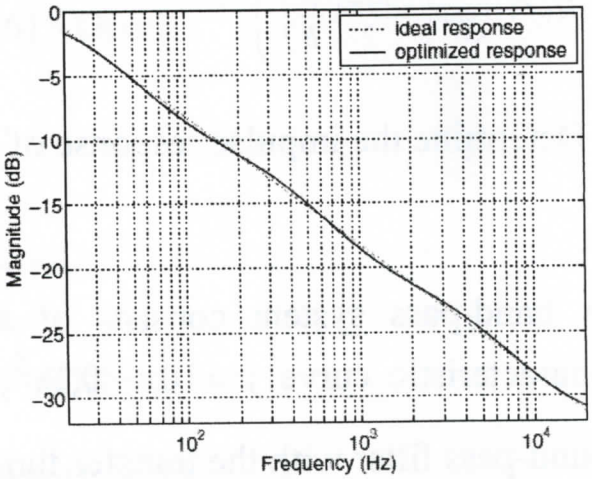
a) Draw the amplitude response of the band-pass filter, and put also the lower and upper frequency limits of the filter into the figure.

b) What is the output signal expression, and which are the frequency of the cosine signals in the output?

6.

Pink noise is used e.g. in audio systems as a test signal. It can be generated by filtering white noise using the circuit shown below.





Assume that the input signal to the filter is bandwidth limited white noise. Its power is uniformly distributed between frequencies 100 Hz - 10000 Hz. The power spectrum (W/Hz) of the bandwidth limited white noise process is given by

$$S(f) \approx \begin{cases} 10^{-3} & 100 \text{ Hz} \le f \le 10 \text{ kHz} \\ 0 & \textit{otherwise} \end{cases}$$

The frequency response function of the filter for the bandwidth of interest is approximately given by

$$H(f) \approx \frac{4}{\sqrt{f}}$$

Determine the mean power of the generated pink noise signal

Hint: $\int \frac{1}{x} dx = \ln x + C$