


Ke-90.2500. Basics in production planning and control.
9.04.2014

1. Answer shortly the following questions:

- Please explain shortly how the optimality conditions of optimization problems can be used. Give an example of an optimization problem type for which no optimality conditions exist.
- What are the shadow prices in linear programming? What are the main goals of the sensitivity analysis in linear programming?
- Can the simplex method be adapted to the general nonlinear programming optimization? Explain your answer.
- Which type of problems is easier to solve: linear programming or integer programming? Explain your answer.
- Please describe briefly different optimality criteria used in production scheduling. What Johnson's method is used for?

 **Simplex method**

a) Simplex method in the standard form

Solve the following problem using the simplex method:

$$\begin{aligned} \max Z &= 5x_1 + 4x_2 - x_3 \\ x_1 + x_2 &\leq 2 \\ x_1 - x_3 &\leq 1 \\ x_1 + x_2 + x_3 &\leq 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

b) The big M method

Consider the following problem:

$$\min Z = 3x_1 + 2x_2 + 4x_3$$

subject to:

$$2x_1 + x_2 + 3x_3 = 60$$

$$3x_1 + 3x_2 + 5x_3 \geq 120$$

$$x_1, x_2, x_3 \geq 0$$

Using big M method construct the complete first simplex tableau for the simplex method. Do not solve the problem!

3. Nonlinear programming and network optimization

a) Nonlinear programming

Consider the following nonlinear programming problem with an equality constraint:

$$\text{minimize } f(x_1, x_2, x_3) = 4x_1^2 + 5x_2^2 + x_3$$

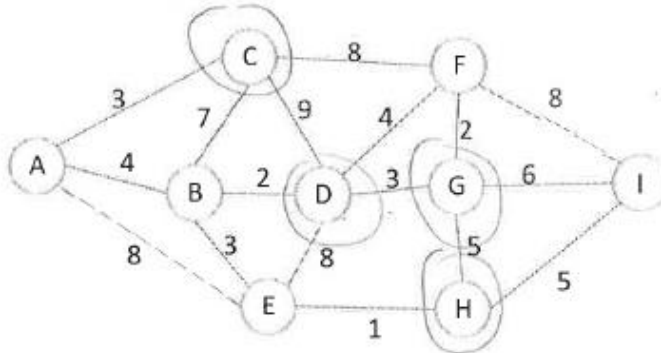
subject to:

$$x_2 + 3x_3 = 6$$

Write down the Lagrange function and obtain the resulting system of equations. You are not required to solve the system of equations!

~~4~~ **Nonlinear programming**

Obtain the minimum spanning tree for the following network:



4. Transportation simplex

A company produces trucks that then are shipped to three distribution centers. The production costs are the same at the two plants (A and B), and the cost of shipping of each truck is shown in the table:

		Distribution center		
		1	2	3
Plant	A	800	700	400
	B	600	800	500

A total of 60 trucks must be produced and shipped, and each distribution center must receive exactly 20 trucks. Each plant can produce and ship any amount up to a maximum of 50 trucks, so there is a flexibility on how to divide the total production between the two plants.

Please formulate the task as a transportation problem by introducing a dummy destination. Solve the problem by using the transportation simplex.

~~5~~ **Dynamic programming**

The owner of a chain of three grocery stores has purchased five packages of a product. The estimated profitability of potential sales of the product differs among the three stores. Therefore, the owner wants to know how to allocate five packages to the three stores to maximize the expected profit. For

administrative reasons, the owner does not wish to split packages between stores. The following table gives the estimated expected profit at each store when allocated various numbers of packages.

Packages	Store		
	1	2	3
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

Use dynamic programming to determine how many of the five packages should be assigned to each of the three stores to maximize the total expected profit.