The exam is three hours long and consists of 4 exercises. The exam is graded on a scale 0-30 points, and the points assigned to each exercise are indicated in parenthesis within the text. If necessary, use separate sheets as scratch papers, but write your final answers in the exam paper clearly, and explain your solutions.

Problem 1 (6pt)

Are the following statements true or false? Justify your answers.

(a) Consider the problems (P) $z = \min\{\mathbf{c}^T\mathbf{x} : \mathbf{x} \in X \subseteq \mathbb{R}^n\}$, and ($\overline{\text{P}}$) $\bar{z} = \min\{\bar{\mathbf{c}}^T\mathbf{x} : \mathbf{x} \in \overline{X} \subseteq \mathbb{R}^n\}$. If $\overline{\text{P}}$ is a relaxation of P, then any valid inequality for $\overline{X}$ is valid for X.

(b) The node-edge incidence matrix of any undirected graph $G = (V, E)$ is totally unimodular (the node-edge incidence matrix of G is a $|V| \times |E|$ matrix $[a_{ie}]$ with $a_{ie} = 1$ if node i is an endpoint of edge e, and $a_{ie} = 0$ otherwise).

(c) Let $\boldsymbol{\pi}\mathbf{x} \leq \pi_0$ be a valid inequality for a set $P \subseteq \mathbb{R}^n_+$. If an inequality $\boldsymbol{\mu}\mathbf{x} \leq \mu_0$ with $\mu_0 < \pi_0$ is valid for P, then the inequality $\boldsymbol{\pi}\mathbf{x} \leq \pi_0$ is dominated by $\boldsymbol{\mu}\mathbf{x} \leq \mu_0$.

Problem 2 (6pt)

Consider the following integer programming problem and the optimal tableau below obtained after solving its LP relaxation:

$$
\begin{aligned}
\text{(IP)}\max\ & 8x_1 + 10x_2 + 6x_3 + 6x_4 \\
\text{s.t.}\ & 9x_1 + 8x_2 + 6x_3 + 6x_4 \leq 12 \\
& 12x_1 + 8x_2 + 8x_3 - 6x_4 \leq 14 \\
& 4x_2 + 4x_3 - x_4 \leq 4 \\
& x_1, x_2, x_3, x_4 \geq 0 \text{ integer.}
\end{aligned}
$$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	57/4	25/16	0	15/8	0	17/16	0	3/8
$x_4 =$	1/2	9/8	0	-1/4	1	1/8	0	-1/4
$x_6 =$	8	33/2	0	-1	0	1/2	1	-3
$x_2 =$	9/8	9/32	1	15/16	0	1/32	0	3/16

(a) Derive a Gomory cut from row 1 and rewrite it in terms of the original variables $x_1, \ldots, x_4$.
(b) Derive the same cut using the Chvátal-Gomory procedure.