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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Wednesday May 21st, 2014, 16:00–19:00

Ensure that every answer sheet contains:
- Your name, degree programme, student number
- Course name "T-79.1001 Introduction to Theoretical Computer Science T" and the date "May 21, 2014"
- The total number of answer sheets submitted for grading

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1. Finite state automata ja regular expressions.

   (a) Show that the language $\{w \in \{a,b\}^* \mid w \text{ starts or ends with the substring } abba\}$ is regular by describing it as a regular expression. *5p.*

   (b) Show that the language

   $$\{w \in \{a,b,c\}^* \mid w \text{ does not contain the substring } aa \text{ or the substring } ab\}$$

   is regular by describing it as a finite state automaton. *5p.*

   (c) Design the deterministic finite state automaton with the minimal number of states that accepts the language described by the regular expression $bc(abc \cup bc)^*$. *5p.*

2. Consider the language

   $$L = \{a^n(ca)^m b^{n+1} \mid m \geq 0 \text{ and } n \geq 0\}.$$

   (a) Show that L is not regular. *7p.*

   (b) Design a context free grammar that produces L. *6p.*

   (c) Give parse trees for the strings $cacab$ and $aacabbb$ in your grammar. *2p.*

3. Design a pushdown automaton that decides whether the input belongs to the language

   $$L = \{w \mid w \in \{a,b\}^*,\ w = w^R\},$$

   where w^R is obtained by taking the characters in w in reverse order. Is your automaton deterministic or nondeterministic? Present the computation of your automaton with inputs ϵ, ab, and $abba$. *15p.*

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4. Let L_1 and L_2 be languages over an alphabet Σ.

(a) Show that if the language L_1 is regular and L_2 is context-free, then the language $\overline{L_1}L_2 = \{xy \in \Sigma^* \mid x \notin L_1 \text{ and } y \in L_2\}$ is also context-free. *5p.*

(b) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language. Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.) *5p.*

(c) Show that if the language L_1 is recursive and L_2 is recursively enumerable, then the language $L = \overline{L_1} \cap L_2$ is recursively enumerable (where $\overline{L_1} = \{x \in \Sigma^* \mid x \notin L_1\}$ is the complement of the language L_1). *5p.*

Total 60p.