T-79.5207 Advanced Course in Algorithms (5 cr) P

Exam, 26 May 2014, 16:00–19:00

Write down on each answer sheet:
– Your name, degree programme, and student number
– The text: "T-79.5207 Advanced Course in Algorithms 26.5.2014"
– The total number of answer sheets you are submitting for grading

Remark: You can write down your answers in either Finnish, Swedish, or English.

1. *Linear programming and approximation algorithms.*
   1. Formulate the SET COVER problem as an integer programming problem.
   2. Formulate the dual of the SET COVER integer program and propose a combinatorial interpretation of this dual IP.
   3. Propose some reasonable approximation algorithm for this dual IP.
2. *Exact and parameterized algorithms.*
   1. What is meant by problem parameterization and fixed-parameter tractability?
   2. Show that the VERTEX COVER problem below is fixed-parameter tractable with respect to the parameter k. Carefully justify your answer.
      Hint: One possible way to approach the problem is to reduce out isolated vertices and vertices of degree more than k.

VERTEX COVER
Input: An undirected graph G with n vertices, an integer $k \geq 0$.
Question: Is there a subset $S \subseteq V(G)$ of vertices with $|S| \leq k$ such that every edge of G has at least one end-vertex in S?

---

3. *Meeting in the middle.* Design an algorithm with running time $O(2^{m/2}mn)$ for the following problem.

EXACT COVER
Input: A family $\mathcal{F}$ consisting of m subsets of $[n] = \{1, 2, \ldots, n\}$.
Question: Do there exist sets $A_1, A_2, \ldots, A_k \in \mathcal{F}$ such that $\cup_{i=1}^{k} A_i = [n]$ and $A_i \cap A_j = \emptyset$ for all $1 \leq i < j \leq k$?

---

4. *Randomised and stochastic algorithms.* Consider a lone rook (Finnish "torni") making random moves on an $n \times n$ chessboard, meaning that at each move, the rook chooses one of its permissible next-state squares uniformly at random. Show that for $n \geq 3$ the Markov chain defined by these moves is regular, and determine its stationary distribution.

FB. *Online feedback (+1p).* Please indicate (yes/no) whether you have given (plan to give) online anonymous feedback via the link available in Noppa?

Grading: Each problem 12p, total 48p. (Online feedback +1p.)

Aalto University, Department of Information and Computer Science
Petteri Kaski (+358 50 430 0948)