T-61.5140 **Machine Learning: Advanced Probabilistic Methods**
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Examination, September 1st, 2014 from 13 to 16 o'clock.

In order to pass the course and earn 5 ECTS credit points, you must pass the exam. Results of this examination are valid for one year after the examination date. Information for Finnish speakers: Voit vastata kysymyksiin myös suomeksi, kysymykset ainoastaan englannin kielellä. Information for Swedish speakers: Du får också svara pä svenska, frågorna finns dock endast på engelska.

1. Define the following terms shortly:
a) Bayes's theorem
b) Hammersley-Clifford theorem
c) conditional independence
d) proposal distribution
e) adaptive rejection sampling
f) variational approximation

2. Write the algorithm for Gibbs sampling and write the distributions to sample from in the case of $p(x_1, x_2, x_3, x_4)$.

3. How can you generate samples from a Laplace distribution. Write a sketch of a program generating the samples. You have access to a computer that is able to generate samples from the uniform distribution. Hint: The probability density function for an Laplacian random variable is here: $p(x \mid \mu, b) = \frac{1}{2b} exp^{(-\frac{|x-\mu|}{b})}$.

4. Write the probability $p(\mathbf{x})$ for the finite mixture model of multivariate Bernoulli distributions, name the parts of the mixture model, and derive the E-step and the M-step of the Expectation-Maximization (EM) algorithm. Hint: The probability for a d-dimensional vector of 0-1 data can be calculated with the following equation: $p(\mathbf{x} \mid \theta) = \prod_{i=1}^{d} \theta_i^{x_i} (1-\theta_i)^{1-x_i}$.

5. In a TV game show, the contestant has the chance to win the price if he chooses a right door out of three doors behind which the price is waiting. First, the contestant is asked to select one of the doors. Then, the game show keeper opens another door which does not contain the prize. After that, the contestant is asked if he wants to change his selection of the door (out of the two doors that remain unopened). Finally, the contestant wins the prize if it is found behind the selected door.

Model the domain with random variables *Price, First Selection, Game show keeper opens* with a Bayesian network and write the probability table(s) for the network according to the rules of the game. Calculate the probability of winning the prize if the contestant (a) does NOT change his original selection of the door, and (b) changes his original selection.