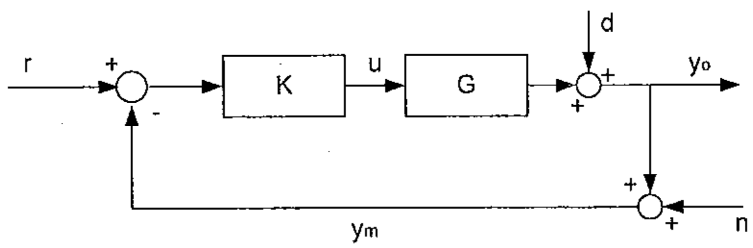
AS-74. 3123 Model-Based Control Systems
Exam 8. 1. 2013

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been accepted.

5 problems.

1. Consider the multivariable closed-loop control configuration



Write the equations describing the system and identify

- **a.** closed-loop transfer function
- **b.** sensitivity function
- **c.** complementary sensitivity function

Write expressions for the output variable y_0, control variable u and error variable $e = r\text{-}y_0$. What conditions to the above functions (a-c) should be set, in order the system to operate "well"?

2. For the system

$$\dot{x}(t) = x(t) + u(t), \;\; x(0) = x_0$$

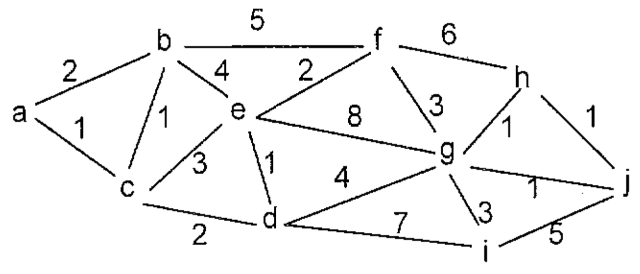
calculate the control law and optimal cost, when the criterion to be minimized is

$$J = \frac{1}{2}x(1)^2 + \int_0^1 u^2(\tau)d\tau$$

3. Consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

- **a.** Calculate the *singular values* of the matrix.
- **b.** If the matrix A describes the input-output behavior of a system, describe the implications of the singular values in terms of system performance.

4. The numbers in the below figure denote costs when moving from one node to another. Movement is possible only from the left to the right.



a. Determine the minimum cost path for movement from node a to node j.
b. Define the *target set* S as $\{h, \mathrm{i}, j\}$. Determine the minimum cost path from node a to the target set S.

5. Consider a multivariable system with the transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{2}{s+1} & \dfrac{3}{s+2} \\ \dfrac{1}{s+1} & \dfrac{1}{s+1} \end{bmatrix}$$

Calculate the poles and zeros of the system. What implications to closed loop performance can be made based on these?

Some formulas, which might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2}x^T(T)S(T)x(T) + \frac{1}{2}\int_{t_0}^{T}\left(x^T Qx + u^T Ru\right)dt$$

$$S(T) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \; t \leq T, \text{ boundary condition } S(T)$$

$$K = R^{-1}B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$