AS-74.3123 Model-based control systems
Exam 23.5.2013

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been accepted.

5 problems.

1. Explain briefly the following concepts

- Principle of Optimality
- Dynamic programming
- Waterbed effect
- Robust stability
- Small gain theorem
- Internal model control

2.a. Draw a schema of the "one-degree-of-freedom" control configuration. Define the concepts *sensitivity function* and *complementary sensitivity function* for it.

2.b. Derive the following relationships (in the formulas S and T are the sensitivity and complementary sensitivity functions, G is the process transfer function matrix, K is the compensator and L the open loop transfer function matrix; the dimensions are assumed to be appropriate)

$$S + T = I$$
$$(I + L)^{-1} L = L(I + L)^{-1}$$
$$G(I + KG)^{-1} = (I + GK)^{-1} G$$
$$T = (I + L^{-1})^{-1}$$

2.c. Consider a SISO-case. Determine the region in the complex plane where $|S| > 1$

How can the result be explained in view of control performance?

3. Consider a SISO-process with the transfer function

$$G(s) = \frac{s + \frac{1}{T_1}}{\left(s + \frac{1}{T_2}\right)\left(s + \frac{1}{T_3}\right)}, \quad T_1 < 0, \quad T_2 < 0, \quad T_3 > 0$$

Explain, what kind of fundamental limitations on control performance can be stated for this system? (Present also some calculations; the formulas in the end of the problems can be of help.)

4. Consider a MIMO system with the transfer function matrix

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

a. What is meant by the *poles* and *zeros* of the system?
b. Determine the poles and zeros of the above system.
c. What conclusions can be made with respect to control?
d. What is meant by the *Relative Gain Array* (RGA)?
e. Calculate RGA(0) in the above example case.
f. What conclusions can be made with respect to control?

5. Consider the 1.order process $G(s) = \frac{1}{s-a}$, which has a realization

$$\dot{x}(t) = ax(t) + u(t)$$
$$y(t) = x(t)$$

so that the state is directly measurable. It is desired to find a control that minimizes the criterion

$$J = \int_0^\infty \left(x^2 + Ru^2\right) dt \qquad (R > 0)$$

Determine the solution and determine the closed loop state equation. Is the closed loop system stable, when the process is i. stable, ii. unstable?

Some formulas that might be useful:

$$\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum_{i=1}^{M} \mathrm{Re}\,(\mathrm{p_i})$$

$$|W_T(p_1)| \le 1 \quad \Rightarrow \quad \omega_0 \ge \frac{p_1}{1 - 1/T_0}$$

$$|W_S(z)| \le 1 \quad \Rightarrow \quad \omega_0 \le \left(1 - 1/S_0\right) z$$

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2} x^T(T) S(T) x(T) + \frac{1}{2} \int_{t_0}^{T} \left(x^T Q x + u^T R u \right) dt$$

$$S(T) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1} B^T S + Q, \quad t \le T, \quad \text{boundary condition } S(T)$$

$$K = R^{-1} B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$