Quantum Physics TFY-3.4323
Whole course examination, 22 May 2014
Please write your answers either in English, Finnish, Swedish or German.
No calculators allowed.

1. Explain briefly the following concepts. (You may use equations if you find it convenient, but it is not necessary.)
   a) Rotating wave approximation.
   b) CNOT gate in the context of quantum information.
   c) Behaviour of quasiparticles near the Fermi surface in Fermi liquid theory.

2. The Hamiltonian in the field operator formulation is

$$\begin{aligned} H &= \int d^3x \left(\frac{\hbar^2}{2m} \nabla\psi^\dagger(\mathbf{x}) \nabla\psi(\mathbf{x}) + U(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \right) \\ &+ \frac{1}{2} \int d^3x \int d^3x' \, \psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')V(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}), \end{aligned}$$

   where U is an external potential, m the mass of the particle, and V the interaction potential. Starting from the Heisenberg equation of motion

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{x},t) = -[H, \psi(\mathbf{x},t)] = -e^{iHt/\hbar}[H, \psi(\mathbf{x},0)]e^{-iHt/\hbar}$$

   derive the equation of motion for the field operator:

$$\begin{aligned} i\hbar \frac{\partial}{\partial t}\psi(\mathbf{x},t) &= \left(-\frac{\hbar^2\nabla^2}{2m} + U(\mathbf{x}) \right) \psi(\mathbf{x},t) \\ &+ \int d^3x' \psi^\dagger(\mathbf{x}',t)V(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}',t)\psi(\mathbf{x},t). \end{aligned}$$

   How does the form of this equation relate to the name "second quantization"?

3. Let us consider a Bose gas in a 1D box of length L described by the Hamiltonian

$$H_0 = \sum_{\mathbf{p}} \frac{\mathbf{p}^2}{2m} \hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}}.$$

   Let us switch on interactions

$$\hat{V} = \sum_{\mathbf{kpq}} V(\mathbf{p}-\mathbf{k}) \hat{a}^\dagger_{\mathbf{k}} \hat{a}^\dagger_{-\mathbf{k}+\mathbf{q}} \hat{a}_{-\mathbf{p}+\mathbf{q}} \hat{a}_{\mathbf{p}},$$

   where $V(\mathbf{k}) = V(|\mathbf{k}|)$ is the scattering amplitude. Assume that the state of the system is initially a Fock state

$$|\Psi\rangle = |n_{\mathbf{k}_1}, n_{\mathbf{k}_2}, \ldots\rangle = \prod_{\mathbf{k}} \frac{1}{\sqrt{n_{\mathbf{k}}!}} \left(\hat{a}^\dagger_{\mathbf{k}} \right)^{n_{\mathbf{k}}} |0\rangle,$$

   where $n_{\mathbf{k}}$ is some integer valued function of $\mathbf{k}$. Of course, the total number of atoms in the system is now $N = \sum_{\mathbf{k}} n_{\mathbf{k}}$. Calculate the 1st order correction to the energy due to the interaction term $\hat{V}$ *i.e.* calculate the expectation value $\langle\Psi|\hat{V}|\Psi\rangle$.

TURN THE PAPER!