

## **AS-84.3128 Estimation and Sensor Fusion Methods (4 op)**

Exam 21.10.2014

- You can answer in English or in Finnish
- Exam contains five (5) problems and you should answer to all of them.
- Its prohibited to use any literature in the exam
- Its allowed to use the collection of equations delivered in this exam
- Its allowed to use a calculator

**1.** Answer briefly to the following questions:

- a) What are the maximum likelihood (ML) and maximum a posteriori (MAP) estimators?
- b) When it is required to use an extended Kalman filter?
- c) What is an augmented state and why it is used?
- d) In what cases it is beneficial to use Information filter, in what cases 'normal' Kalman filter? Which one is better in decentralized estimation?
- e) How is Multiple Model Approach different from normal Kalman filter?

(8 p)

**2.** Answer to following questions about the normal (linear) Kalman filter:

- a) Describe what are the presumptions made about the dynamic model, measurements and the associated noises.
- b) In the Kalman filter equations, what describes the accuracy of state estimate?
- c) In the Kalman filter equations, what describes the covariance of measurement residual?
- d) Why a posteriori covariance is always 'smaller' than a priori covariance in Kalman filter?

(6 p)



3. Let there be three measurements

<b>index i</b>	1	2	3
<b>input u(i)</b>	3	2	1
<b>measurement z(i)</b>	1	6	8

The measurement model structure is

$$z(i) = u(i)x_1 + i x_2 + \omega(i), \text{ where } \omega(i) \text{ is an unknown measurement error.}$$

First convert the model to standard least-squares format  $z(i) = H(i)x + \omega(i)$ .

Estimate the two unknown parameters  $x_1$  and  $x_2$  using least squares non-recursive algorithm. All measurements have the same weight.

(6 p)

4. A following continuous time model represents a vehicle moving in 2D.

$$\dot{x}_1 = x_4 \cos(x_3)$$

$$\dot{x}_2 = x_4 \sin(x_3)$$

$$\dot{x}_3 = x_4 \frac{1}{b} \tan(u_1)$$

$$\dot{x}_4 = u_2$$

The state variables of the model are: x-position, y-position, heading angle and the velocity. The controls to the system are: the steering angle and the acceleration of the vehicle.  $b$  is a known parameter defining the wheelbase of the vehicle.

Two radars give a measurement of the squared euclidean distance to the vehicle. The measurement model is as follows:

$$y_1 = x_1^2 + x_2^2$$

$$y_2 = (x_1 - 2)^2 + x_2^2$$

- a) Discretize the continuous time system using Euler method:

$$x(k+1) = x(k) + \Delta T f(x(k), u(k)). \text{ The discretization step } \Delta T = 1s.$$

- b) Calculate the Jacobians that are required for implementing an EKF.

- c) Calculate the Hessians that are required for implementing a second-order EKF.

(6 p)

5. What is an optimal nonlinear estimator? Why, in cases of nonlinear systems, the implemented state estimators are almost always only approximations of the optimal state estimators?

(4 p)